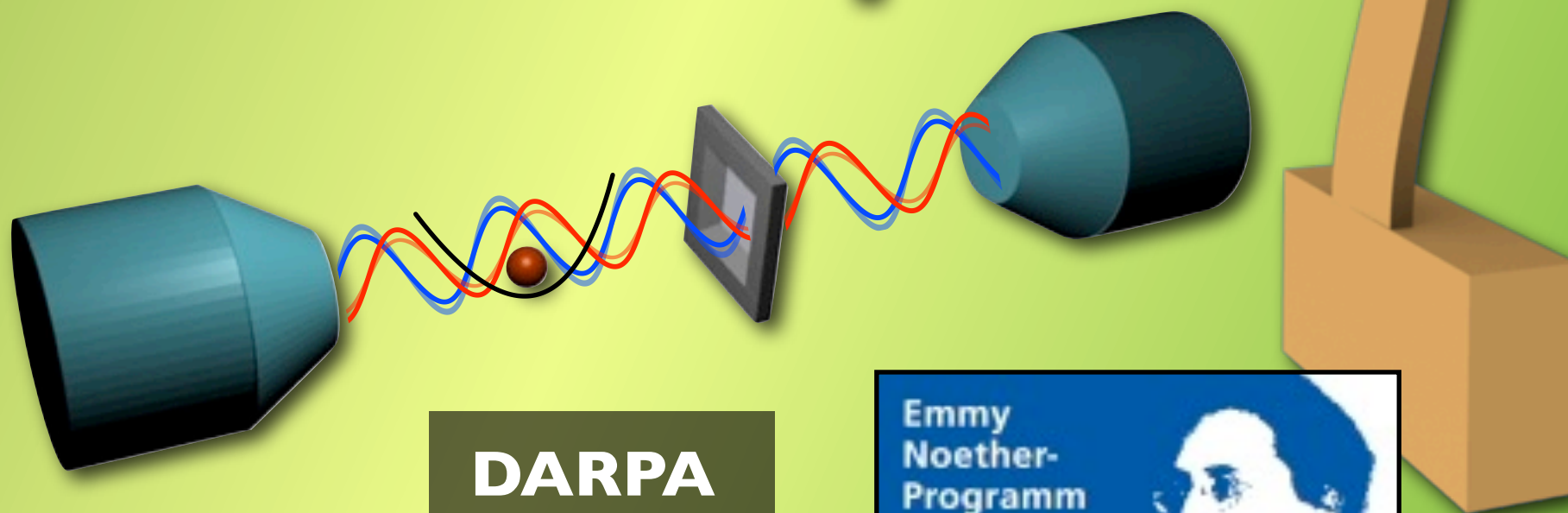


# Cavity optomechanics: – interactions between light and nanomechanical motion

Florian Marquardt

University of Erlangen-Nuremberg, Germany, and  
Max-Planck Institute for the Physics of Light (Erlangen)



**DARPA  
ORCHID**



# Radiation pressure



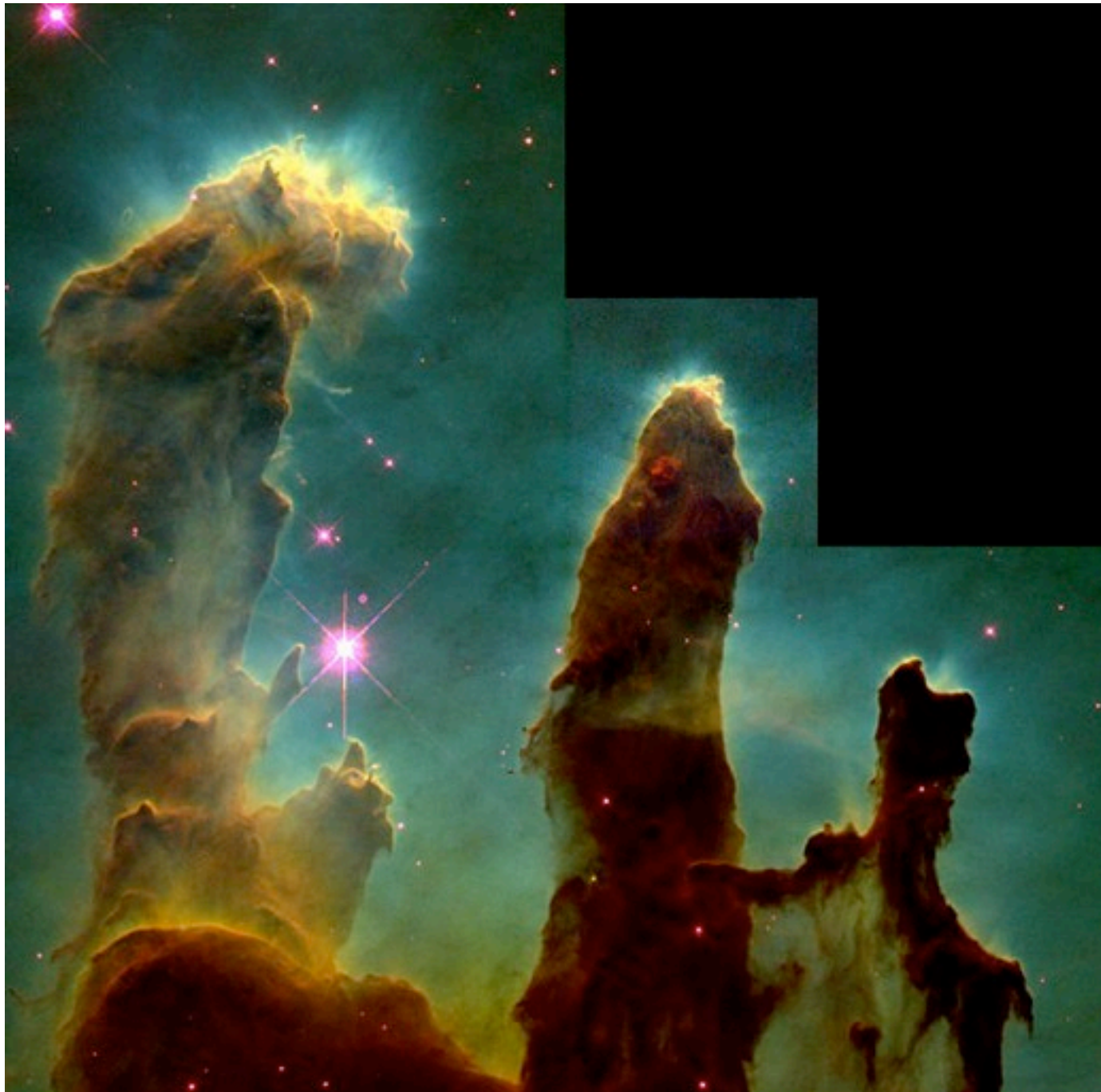
(Comet Hale-Bopp; by Robert Alleva)



**Johannes Kepler**  
De Cometis, 1619



# Radiation pressure



**Johannes Kepler**  
De Cometis, 1619

# Radiation pressure

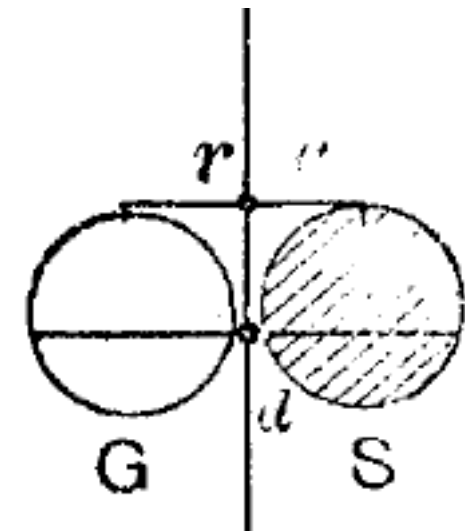
Nichols and Hull, 1901

Lebedev, 1901

## A PRELIMINARY COMMUNICATION ON THE PRESSURE OF HEAT AND LIGHT RADIATION.

BY E. F. NICHOLS AND G. F. HULL.

MAXWELL,<sup>1</sup> dealing mathematically with the stresses in an electro-magnetic field, reached the conclusion that "in a medium in which waves are propagated there is a pressure normal to the waves and numerically equal to the energy in unit volume."

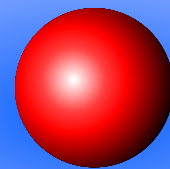


$$F = \frac{2I}{c}$$

Nichols and Hull, Physical Review **13**, 307 (1901)



# Radiation forces



Trapping and cooling

- Optical tweezers
- Optical lattices

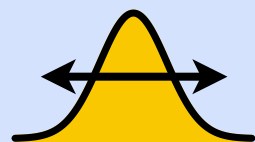
...but usually no back-action from motion onto light!

# Optomechanics on different length scales

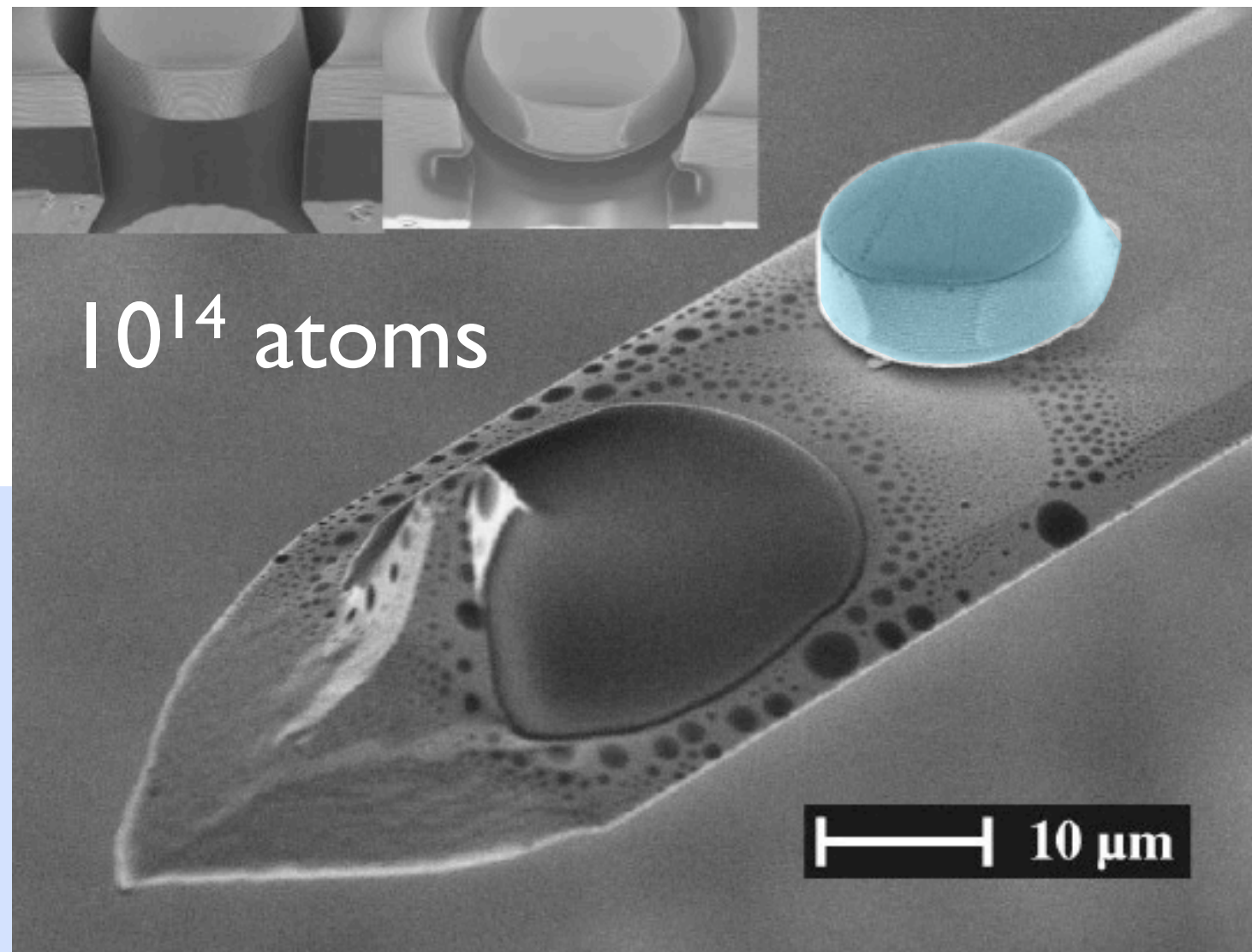


**LIGO – Laser Interferometer  
Gravitational  
Wave Observatory**

$$\omega_M \sim 1\text{kHz} - 1\text{GHz}$$
$$m \sim 10^{-12} - 10^{-10}\text{kg}$$
$$x_{\text{ZPF}} \sim 10^{-16} - 10^{-14}\text{m}$$
$$x_{\text{ZPF}} = \sqrt{\hbar/(2m\omega_M)}$$

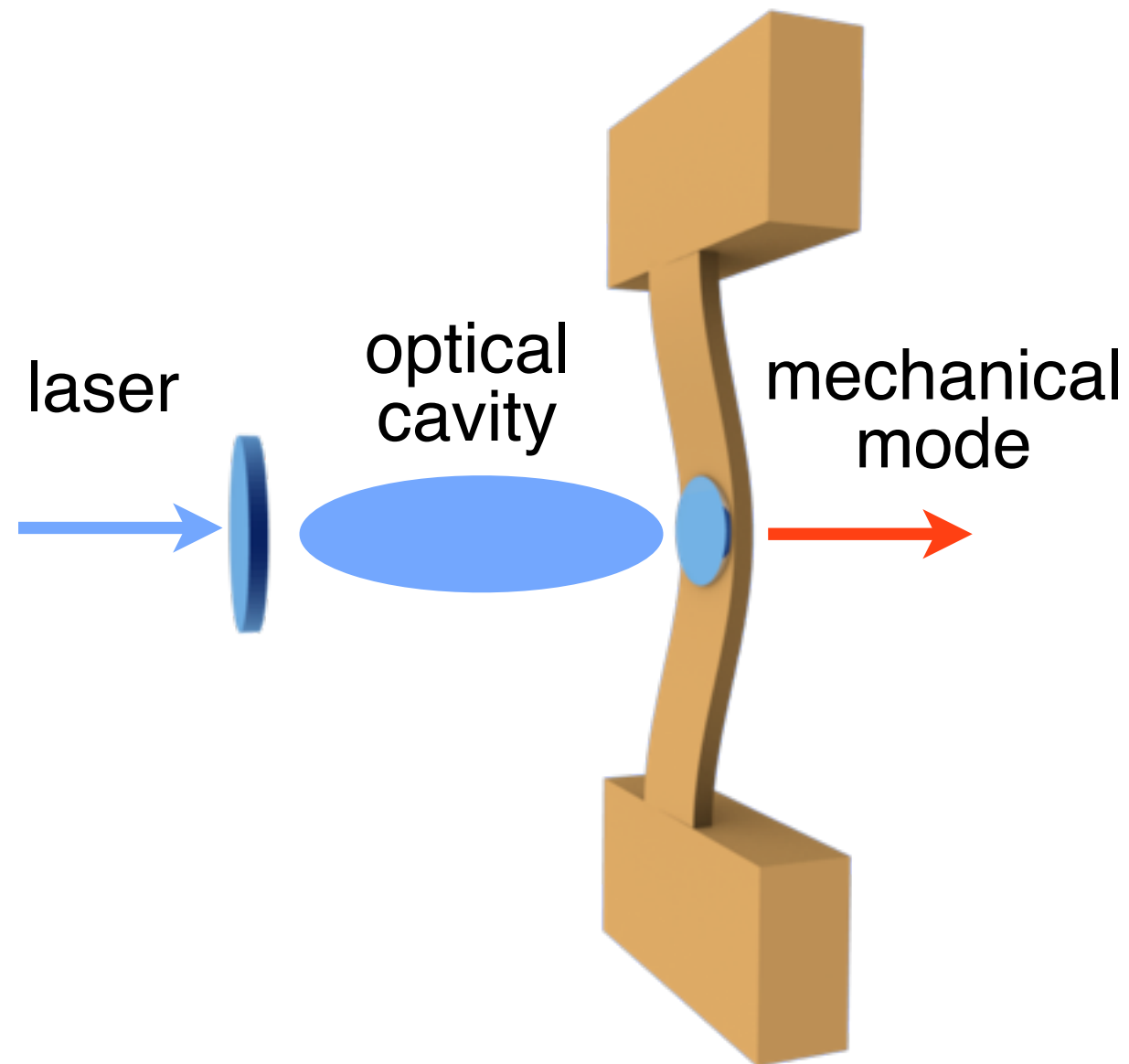


**Mirror on cantilever –  
Bouwmeester lab, Santa Barbara**

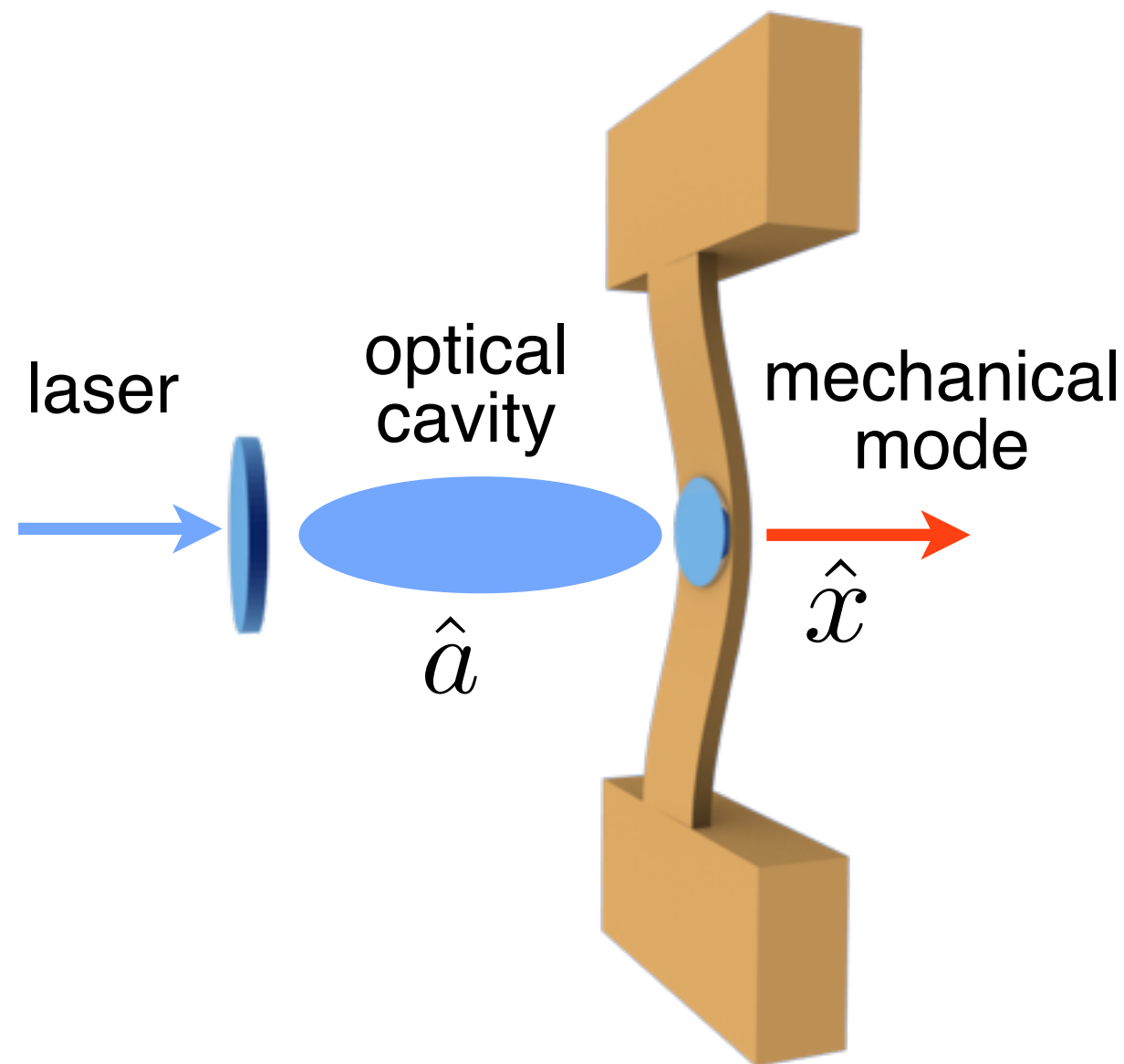




# Optomechanical Hamiltonian



# Optomechanical Hamiltonian

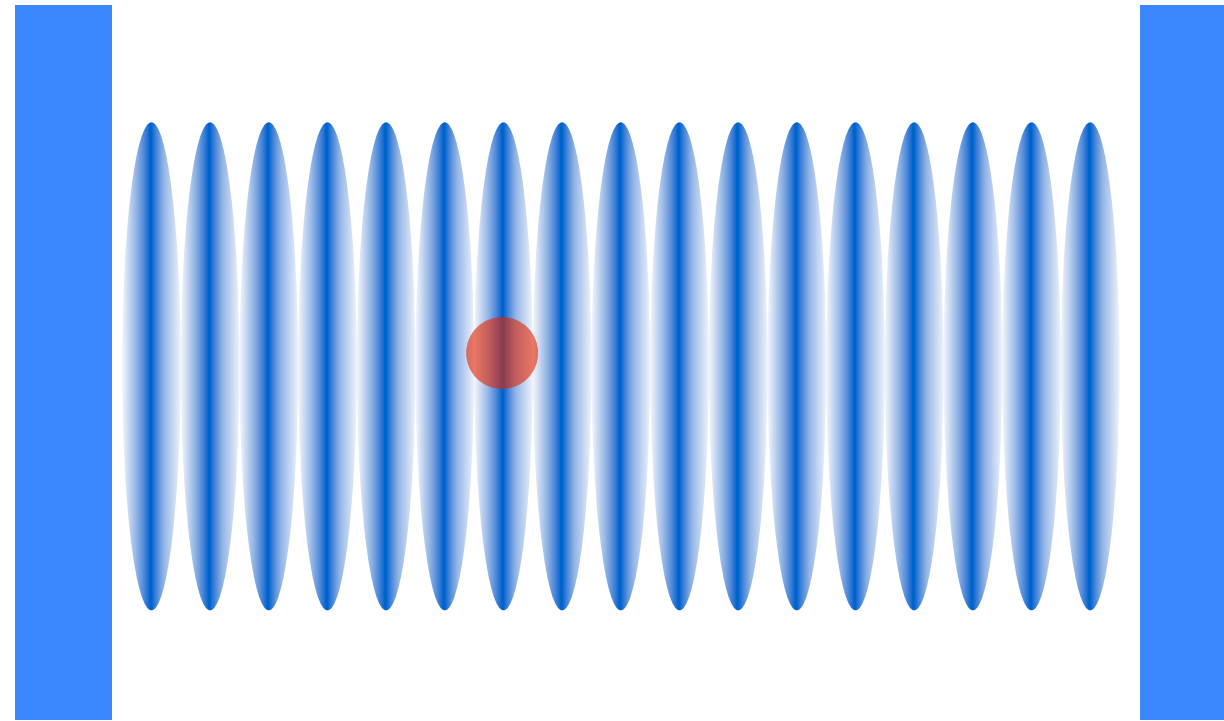


$$\hat{H} = \hbar\omega_{\text{cav}} \cdot (1 - \hat{x}/L)\hat{a}^\dagger\hat{a} + \hbar\omega_M\hat{b}^\dagger\hat{b} + \dots$$

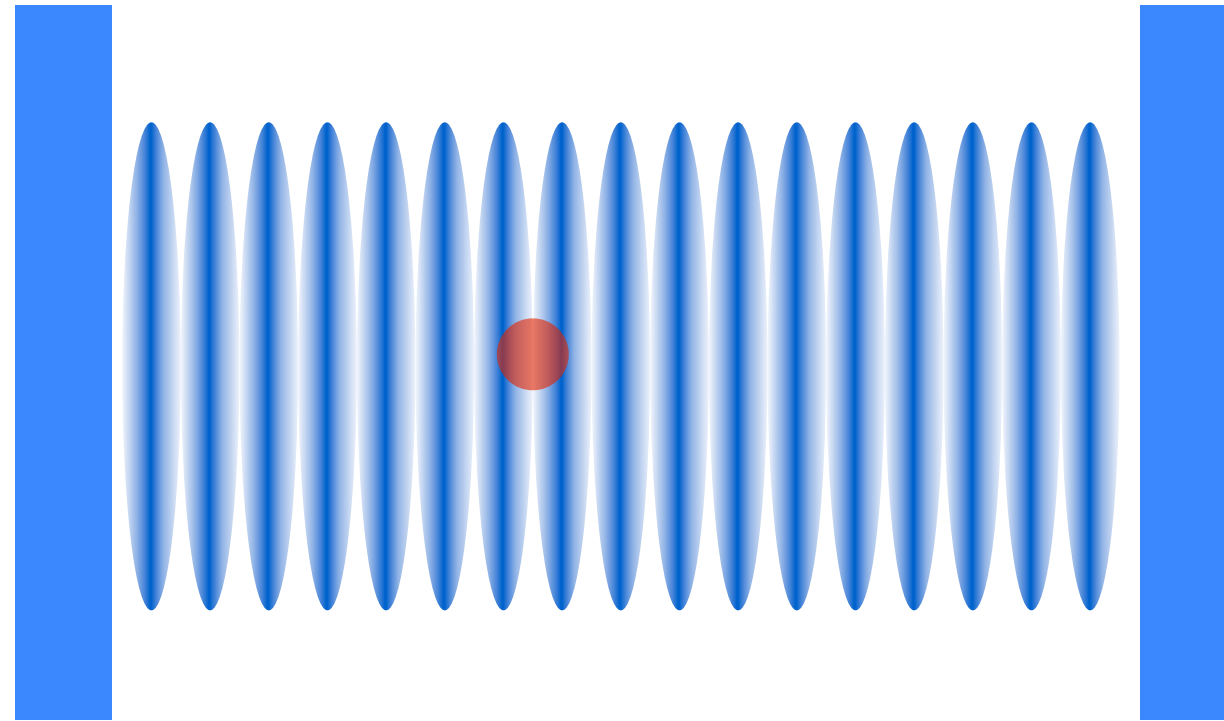
$$\hat{x} = x_{\text{ZPF}}(\hat{b} + \hat{b}^\dagger) \quad x_{\text{ZPF}} = \sqrt{\frac{\hbar}{2m\Omega}}$$



# Optomechanical Hamiltonian

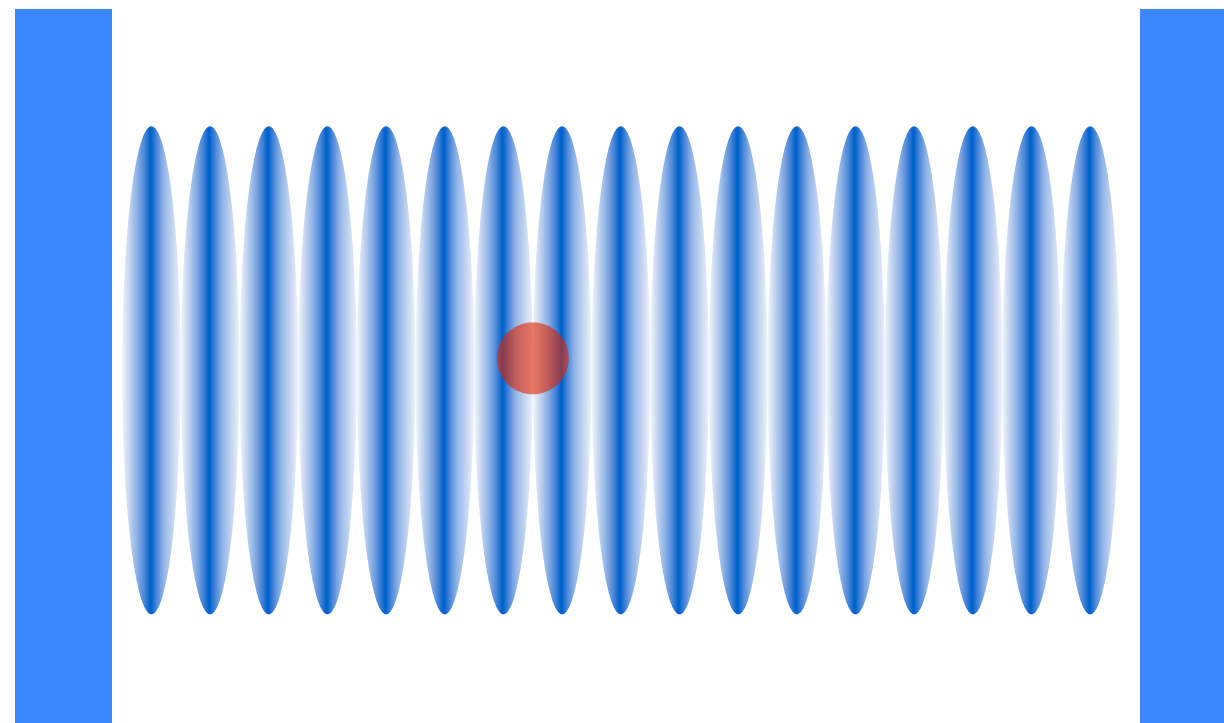


# Optomechanical Hamiltonian





# Optomechanical Hamiltonian

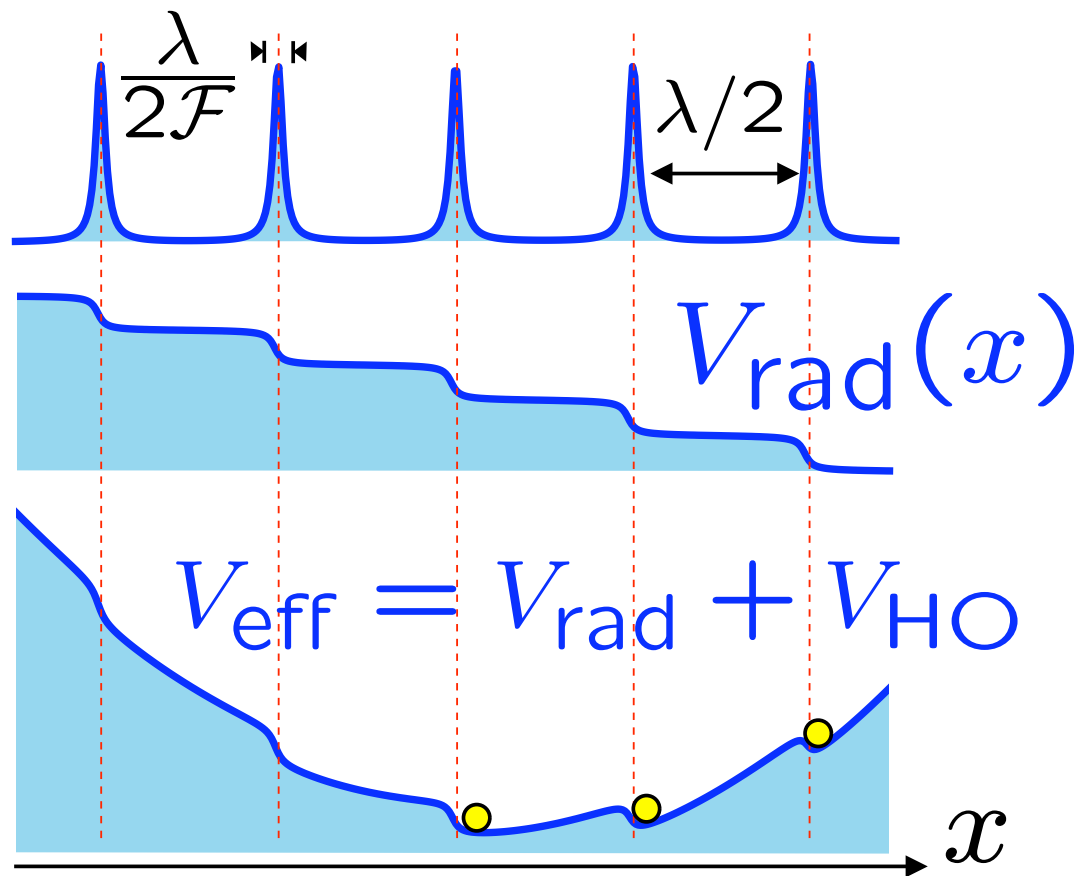


$$\hat{H} = \hbar\omega_{\text{cav}}(\hat{x})\hat{a}^\dagger\hat{a} + \hbar\omega_M\hat{b}^\dagger\hat{b} + \dots$$

...any dielectric moving inside a cavity  
generates an optomechanical interaction!

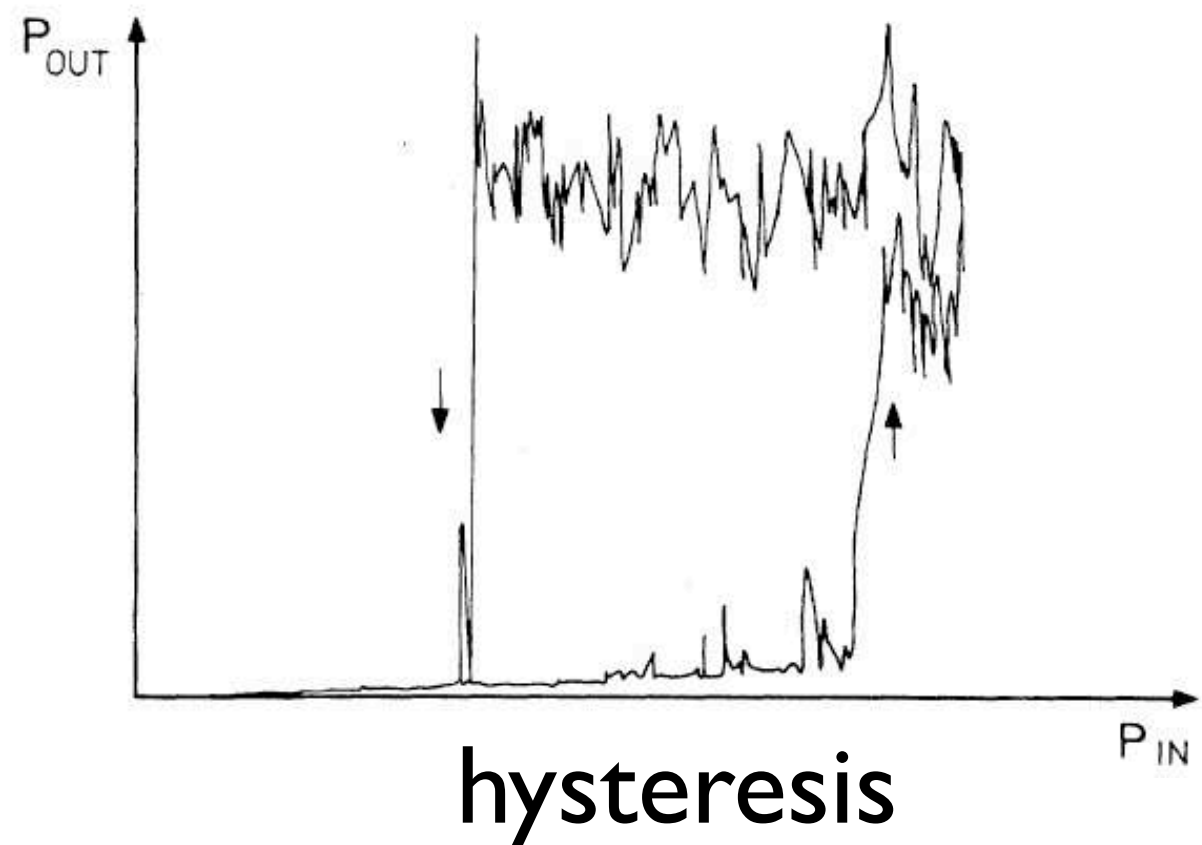
# Basic physics: Statics

$$F_{\text{rad}}(x) = 2I(x)/c$$

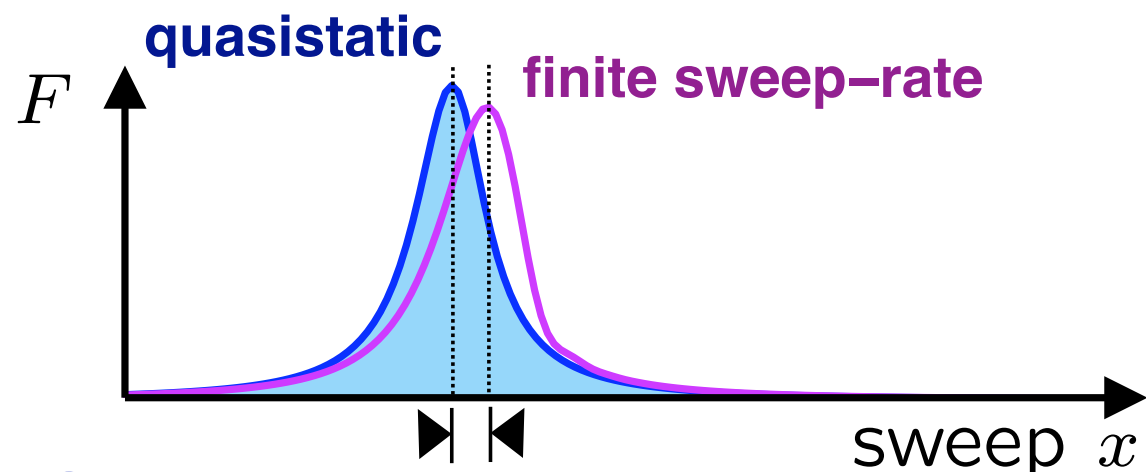


**Experimental proof of static bistability:**

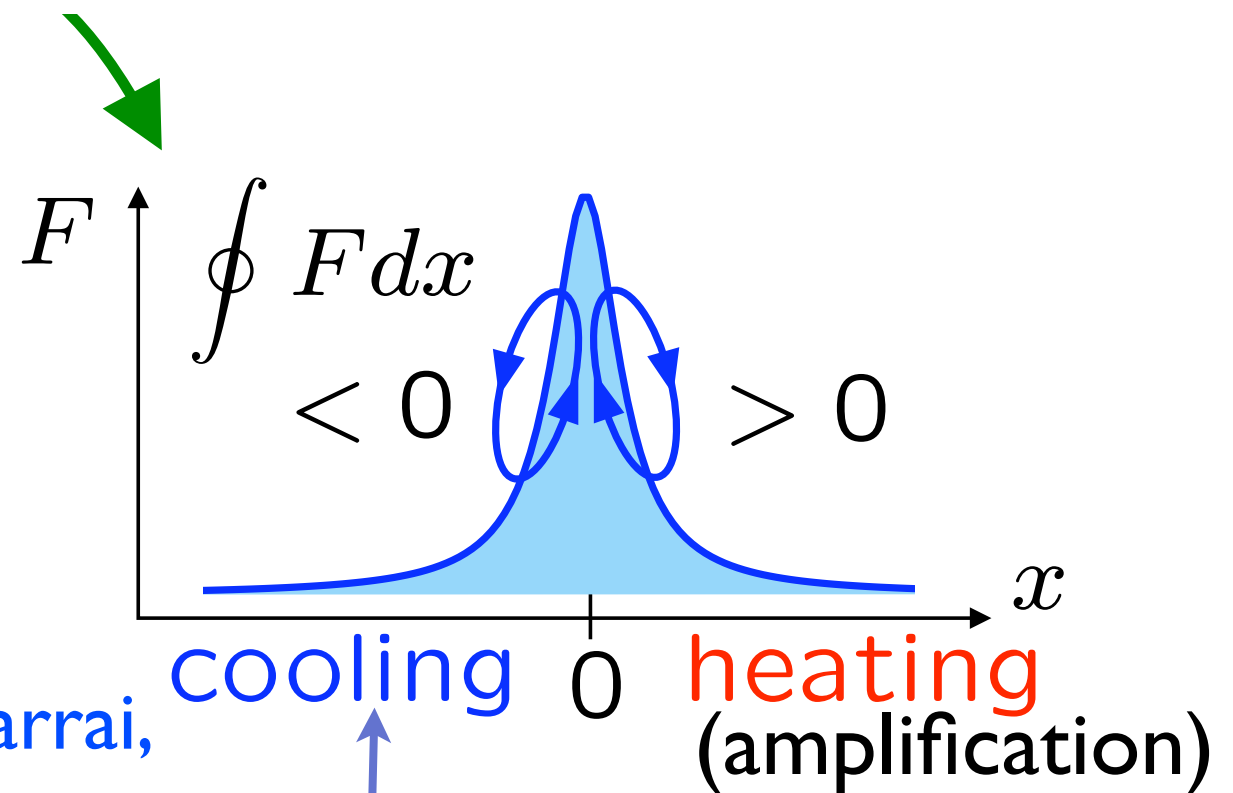
**A. Dorsel, J. D. McCullen, P. Meystre,  
E. Vignes and H. Walther:  
Phys. Rev. Lett. 51, 1550 (1983)**



# Basic physics: dynamics



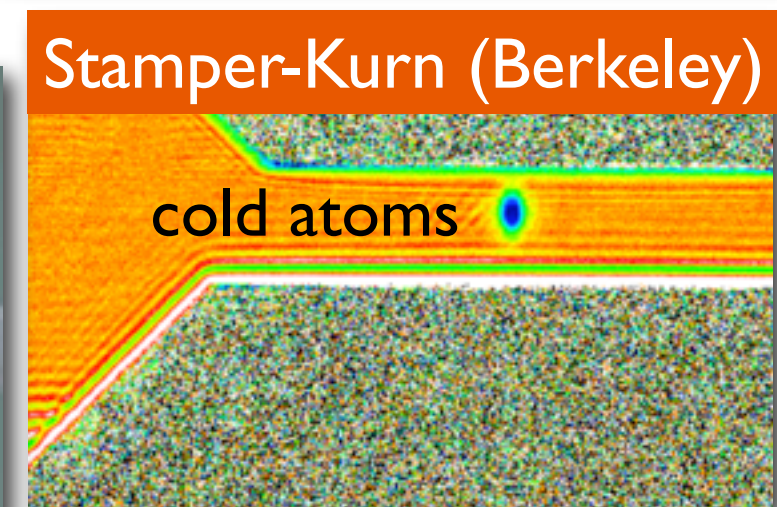
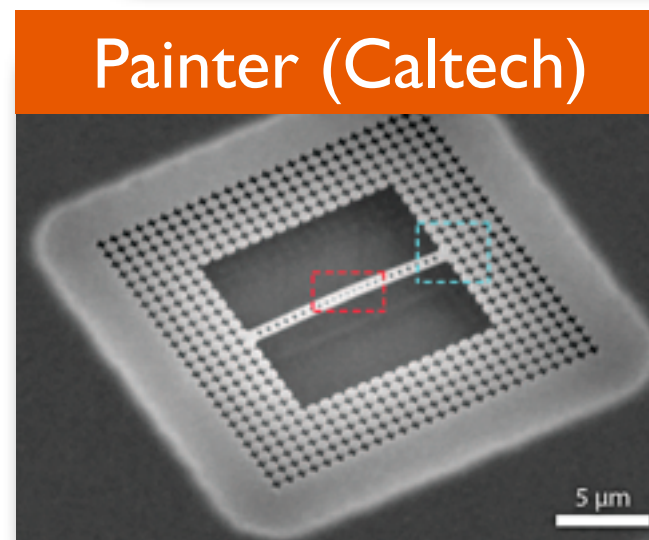
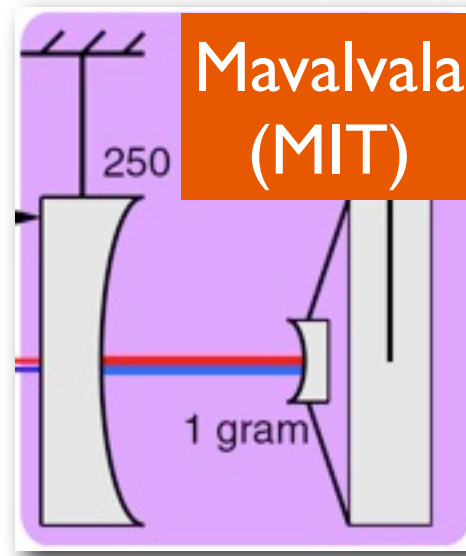
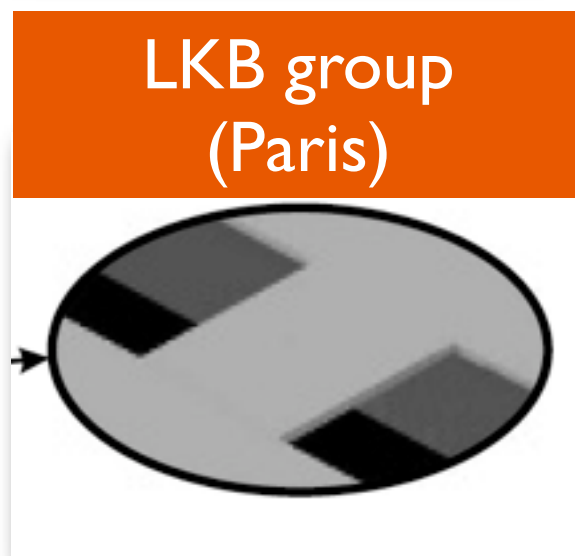
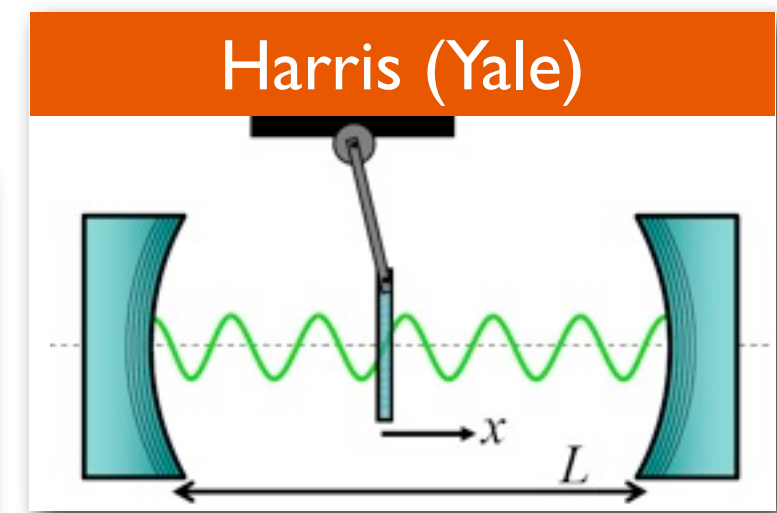
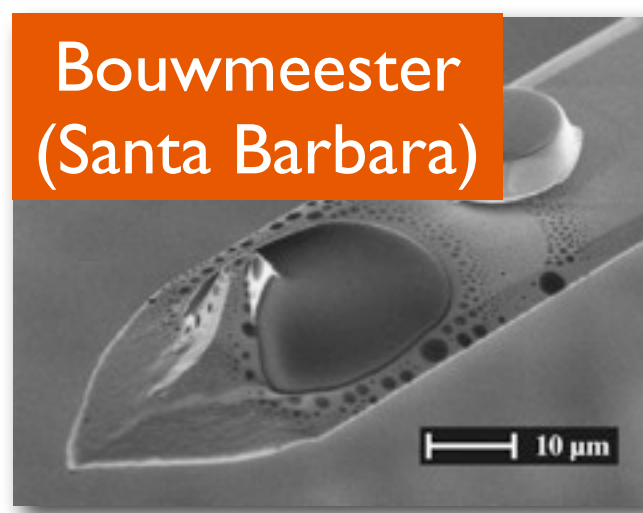
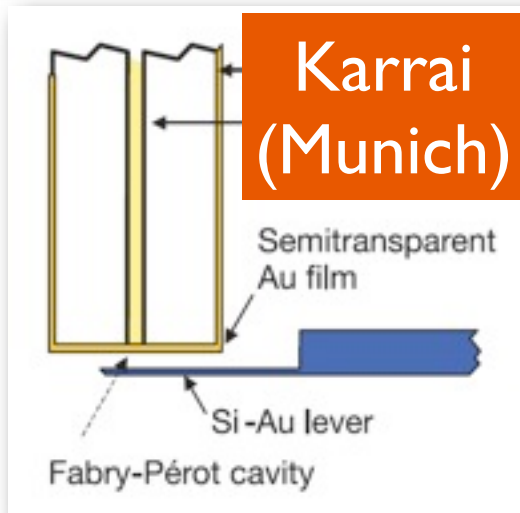
finite optical ringdown time  $\kappa^{-1}$  –  
delayed response to cantilever motion



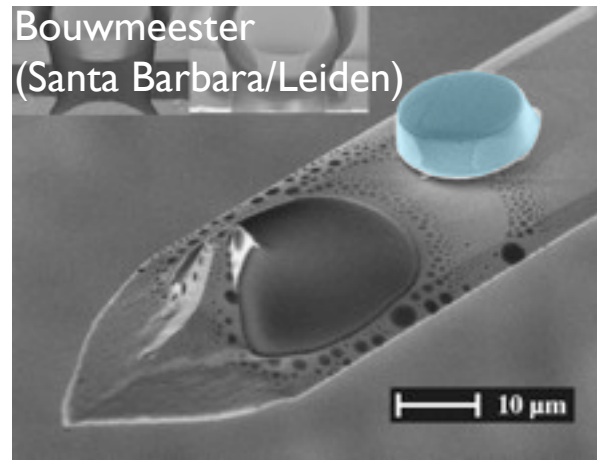
Höhberger-Metzger and Karrai,  
Nature **432**, 1002 (2004):  
300K to 17K [photothermal force]



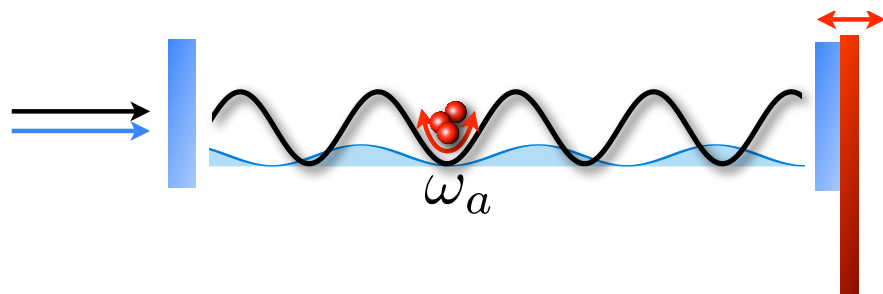
# The zoo of optomechanical (and analogous) systems



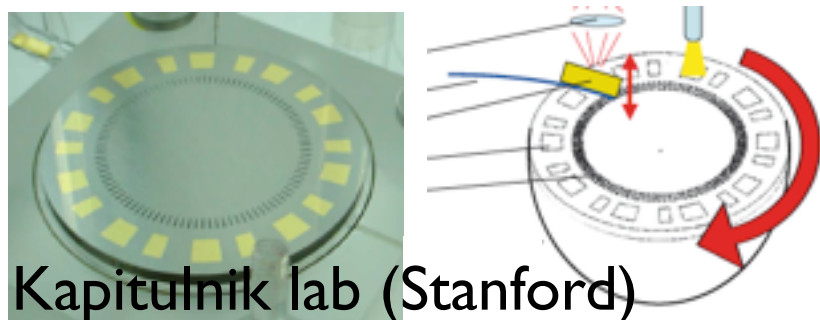
# Optomechanics: general outlook



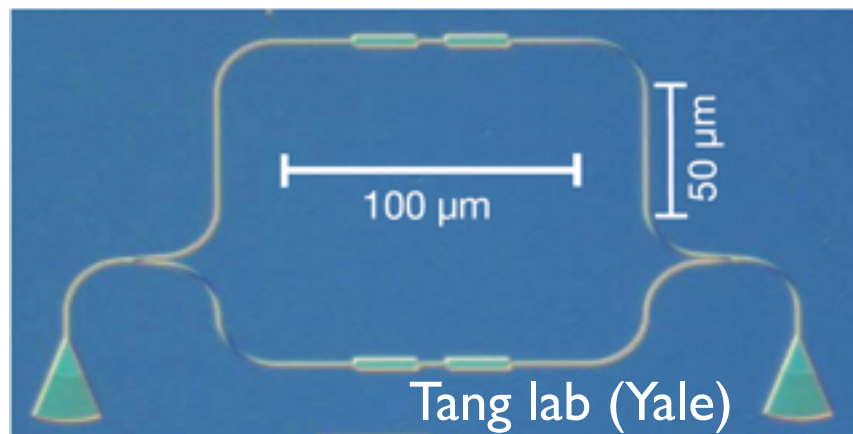
**Fundamental tests of quantum mechanics in a new regime:** entanglement with 'macroscopic' objects, unconventional decoherence? [e.g.: gravitationally induced?]



**Mechanics as a 'bus' for connecting hybrid components:** superconducting qubits, spins, photons, cold atoms, ....



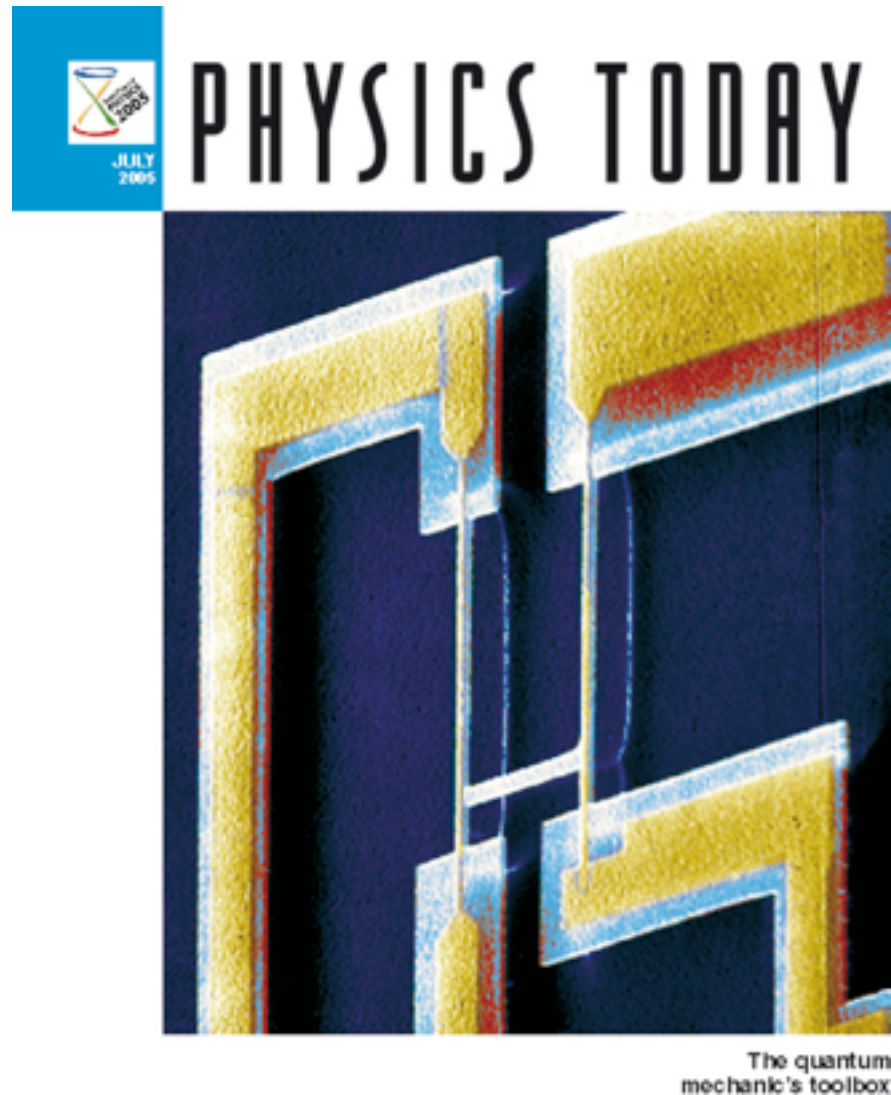
**Precision measurements** [e.g. testing deviations from Newtonian gravity due to extra dimensions]



**Optomechanical circuits & arrays**  
Exploit nonlinearities for classical and quantum information processing, storage, and amplification; study collective dynamics in arrays



# Towards the quantum regime of mechanical motion



## Putting Mechanics into Quantum Mechanics

Nanoelectromechanical structures are starting to approach the ultimate quantum mechanical limits for detecting and exciting motion at the nanoscale. Nonclassical states of a mechanical resonator are also on the horizon.

Keith C. Schwab and Michael L. Roukes

**E**verything moves! In a world dominated by electronic devices and instruments it is easy to forget that all measurements involve motion, whether it be the motion of electrons through a transistor, Cooper pairs or quasiparticles through a superconducting quantum interference device (SQUID), photons through an optical interferometer—or the simple displacement of a mechanical element.

achieved to read out those devices, now bring us to the realm of quantum mechanical systems.

### The quantum realm

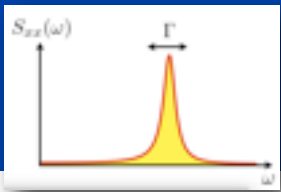
What conditions are required to observe the quantum properties of a mechanical structure, and what can we learn when we encounter them? Such questions have received

**Schwab and Roukes, Physics Today 2005**

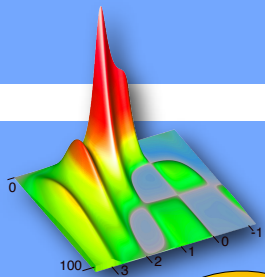
- nano-electro-mechanical systems  
Superconducting qubit coupled to nanoresonator: Cleland & Martinis 2010
- optomechanical systems



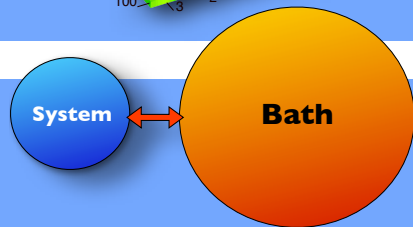
# Optomechanics (Outline)



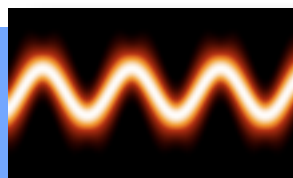
Displacement detection



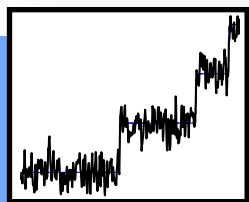
Linear optomechanics



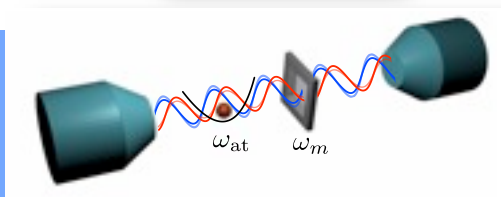
Nonlinear dynamics



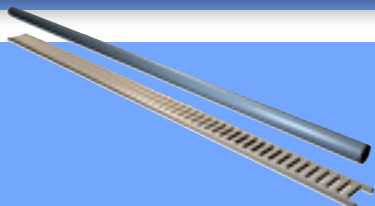
Interesting quantum states



Towards Fock state detection

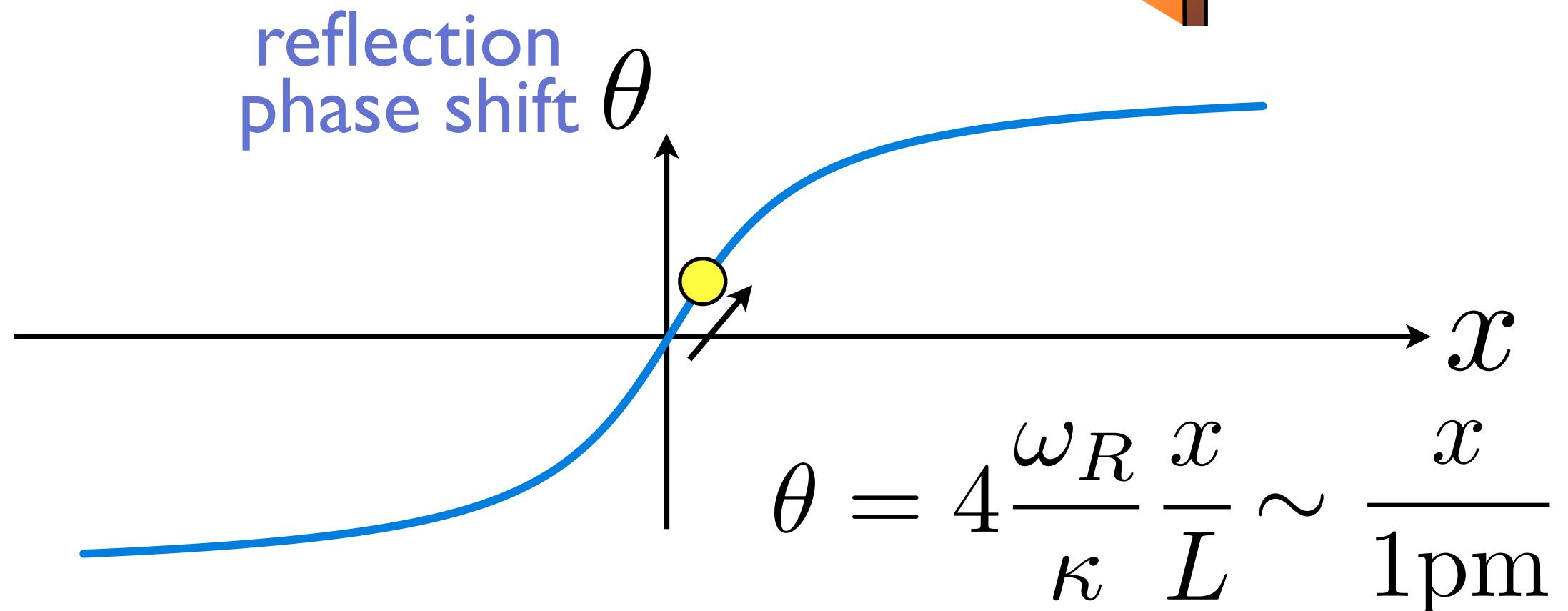
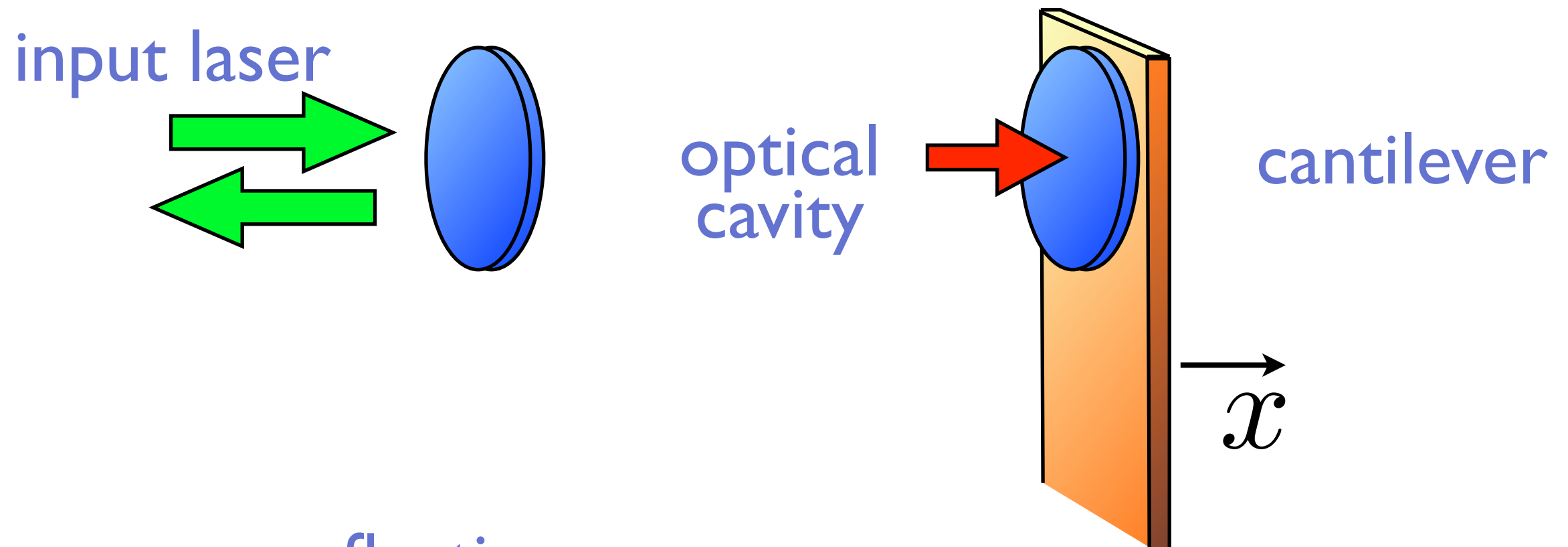


Hybrid systems: coupling to atoms

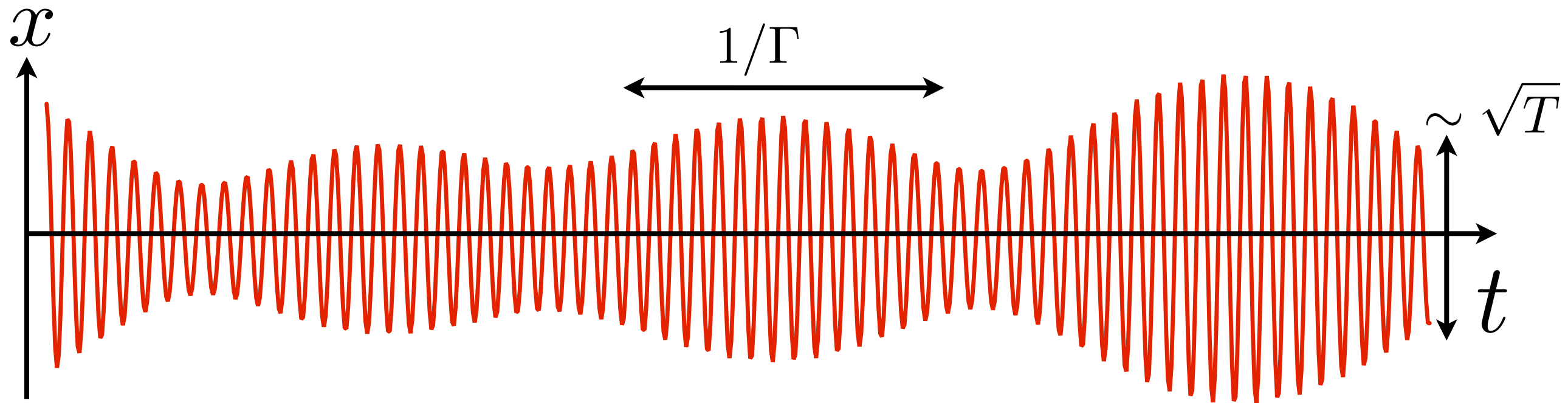


Optomechanical crystals & arrays

# Optical displacement detection



# Thermal fluctuations of a harmonic oscillator



Classical equipartition theorem:

$$\frac{m\omega_M^2}{2} \langle x^2 \rangle = \frac{k_B T}{2} \Rightarrow \langle x^2 \rangle = \frac{k_B T}{m\omega_M^2} \text{ extract temperature!}$$

Possibilities:

- Direct time-resolved detection
- Analyze **fluctuation spectrum of x**



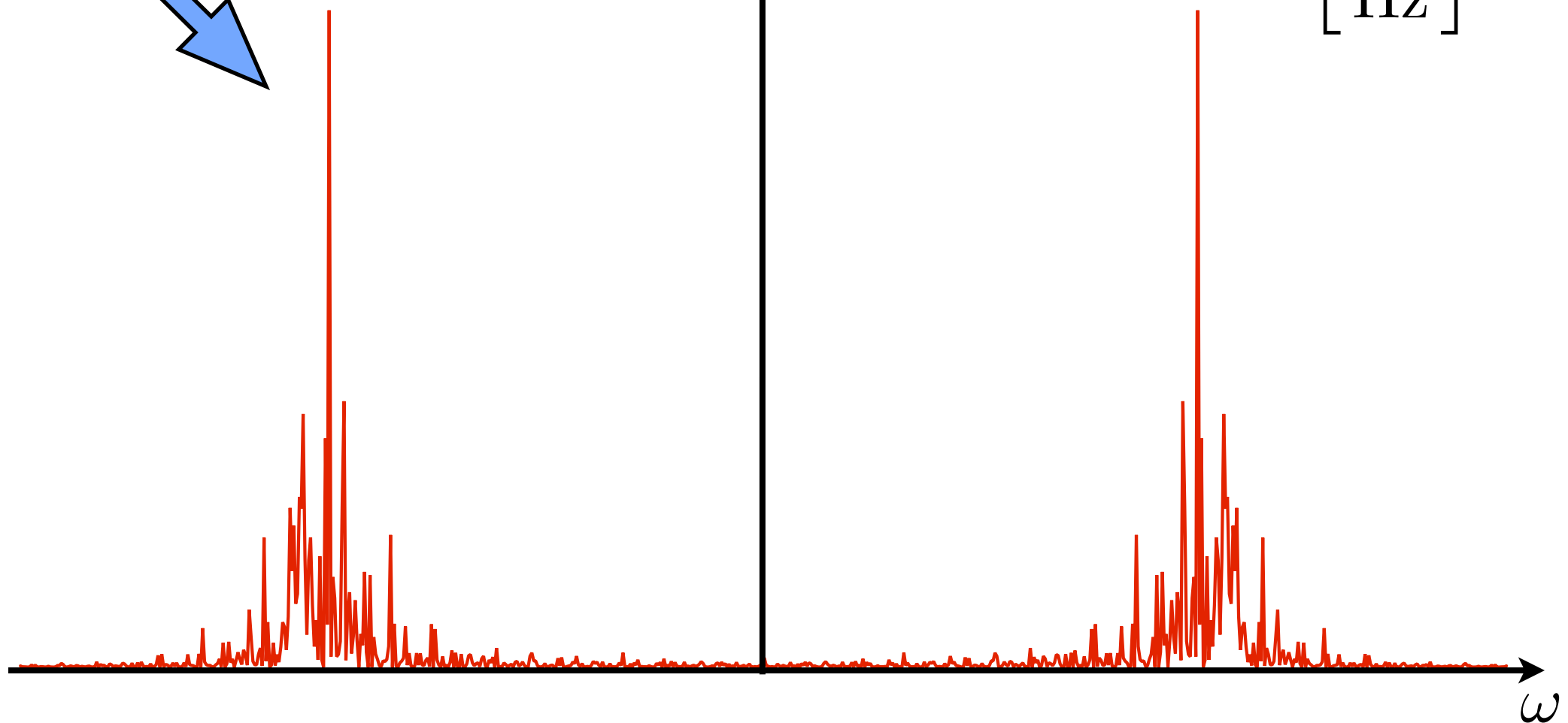
# Fluctuation spectrum

$$\tilde{x}(\omega) = \frac{1}{\sqrt{\tau}} \int_0^\tau dt e^{i\omega t} x(t)$$

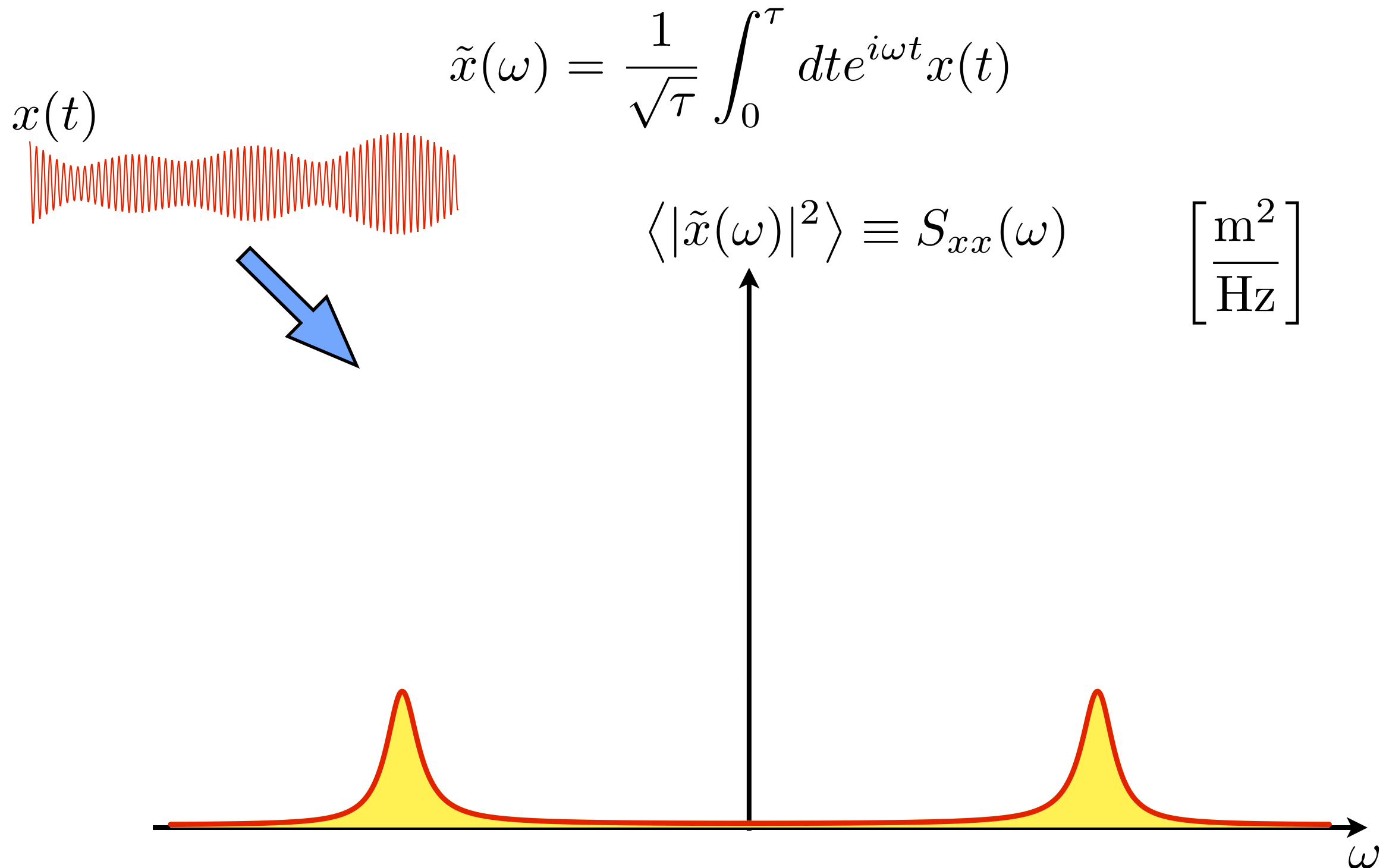
$x(t)$

$|\tilde{x}(\omega)|^2$

$\left[ \frac{\text{m}^2}{\text{Hz}} \right]$



# Fluctuation spectrum



# Fluctuation-dissipation theorem

## General relation between noise spectrum and linear response susceptibility

$$\langle \delta x \rangle(\omega) = \chi_{xx}(\omega) F(\omega)$$

**susceptibility**

$$S_{xx}(\omega) = \frac{2k_B T}{\omega} \text{Im} \chi_{xx}(\omega) \quad (\text{classical limit})$$



# Fluctuation-dissipation theorem

## General relation between noise spectrum and linear response susceptibility

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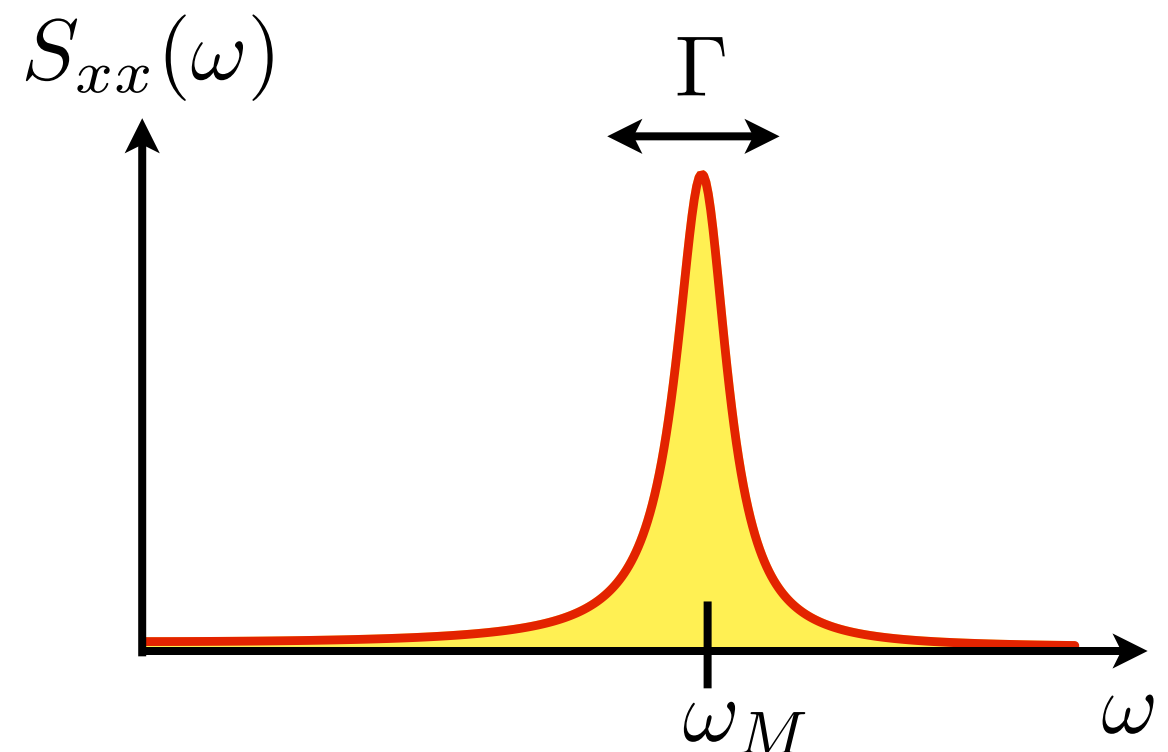
**susceptibility**

$$S_{xx}(\omega) = \frac{2k_B T}{\omega} \text{Im} \chi_{xx}(\omega) \quad (\text{classical limit})$$

for the damped oscillator:

$$m\ddot{x} + m\omega_M^2 x + m\Gamma\dot{x} = F$$

$$x(\omega) = \frac{1}{\underbrace{m(\omega_M^2 - \omega^2) - im\Gamma\omega}_{\chi_{xx}(\omega)}} F(\omega)$$



# Fluctuation-dissipation theorem

## General relation between noise spectrum and linear response susceptibility

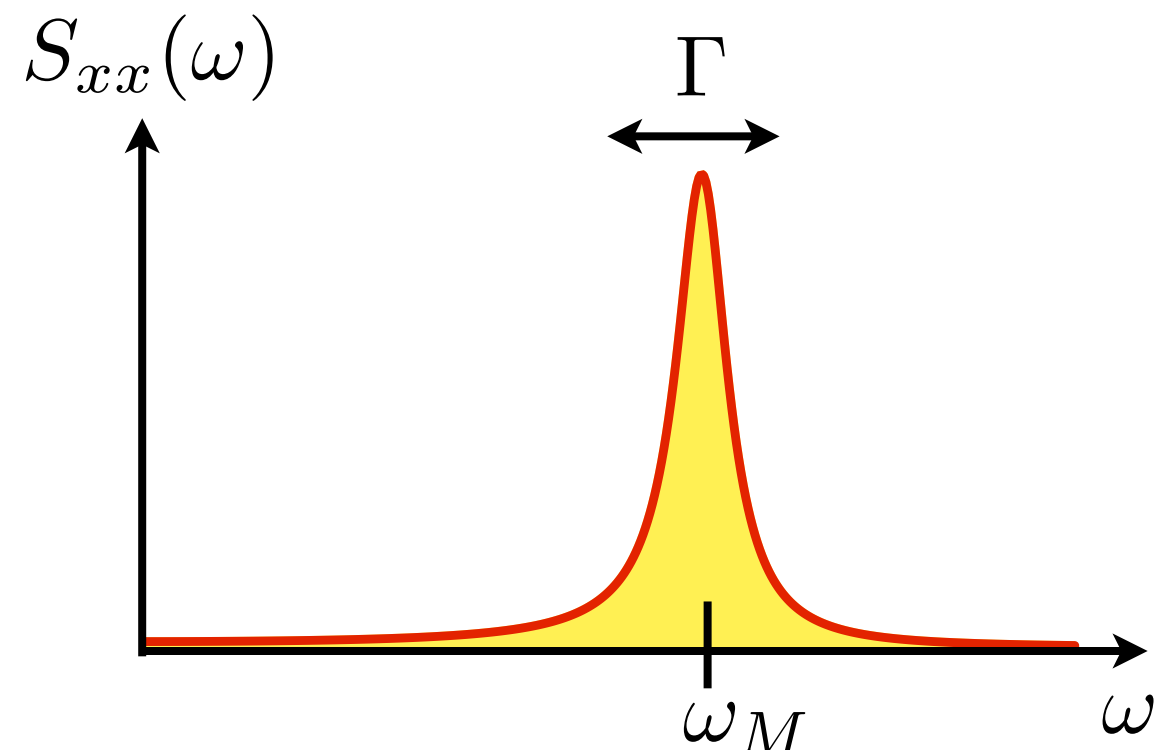
$$\langle \delta x \rangle(\omega) = \chi_{xx}(\omega) F(\omega)$$

**susceptibility**

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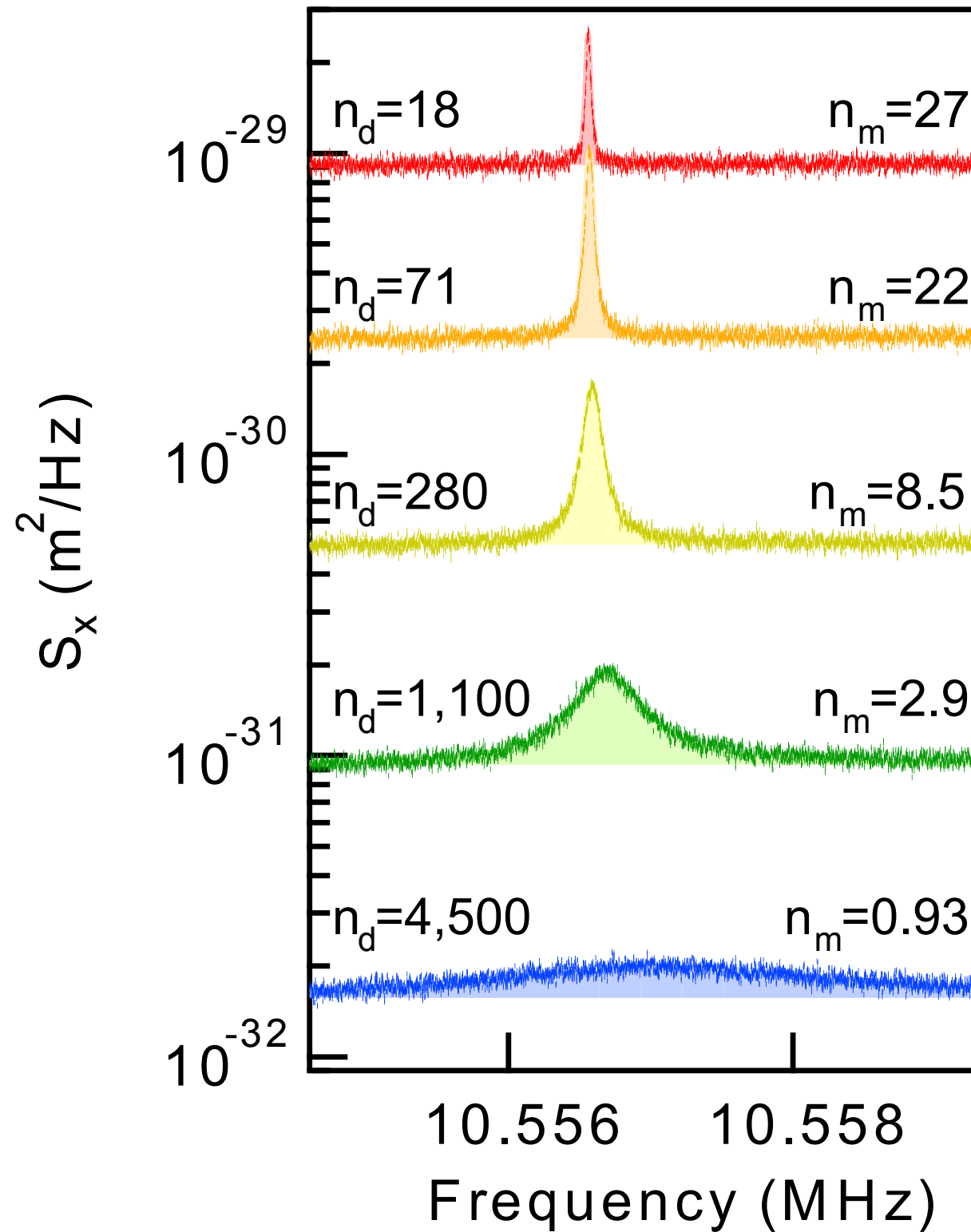


**area yields  
variance of x:**

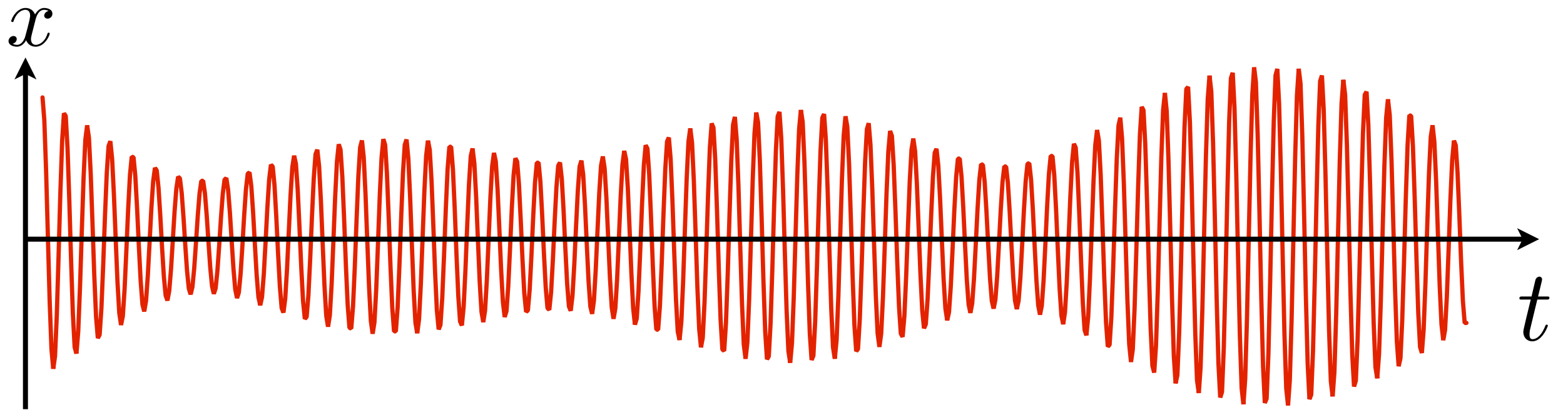
$$\int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} S_{xx}(\omega) = \langle x^2 \rangle$$

**...yields  
temperature!**

# Displacement spectrum

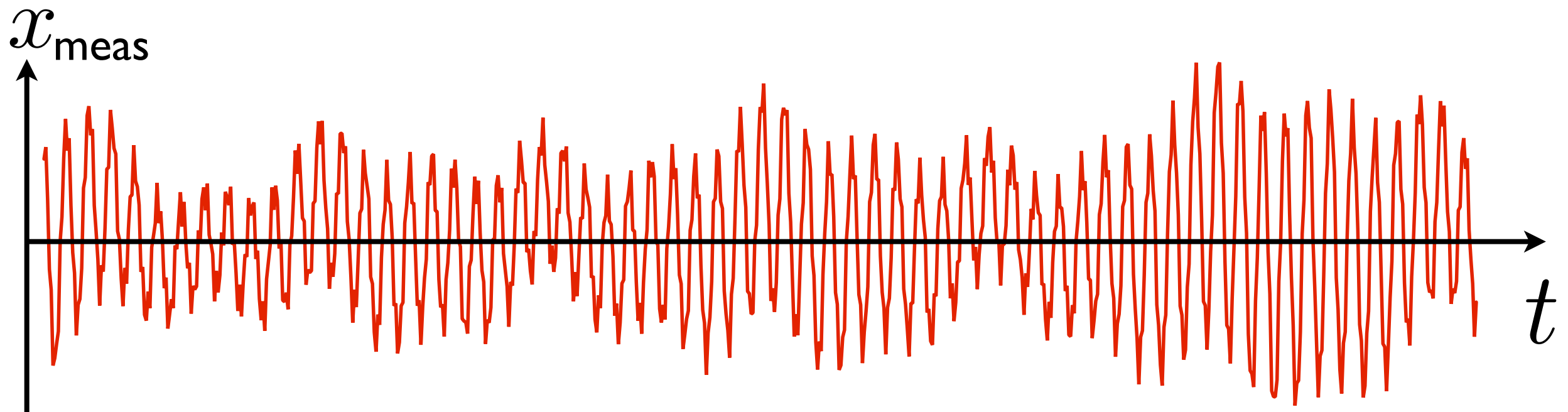


# Measurement noise





# Measurement noise



$$x_{\text{meas}}(t) = x(t) + x_{\text{noise}}(t)$$

Two contributions to  $x_{\text{noise}}(t)$

1. measurement imprecision

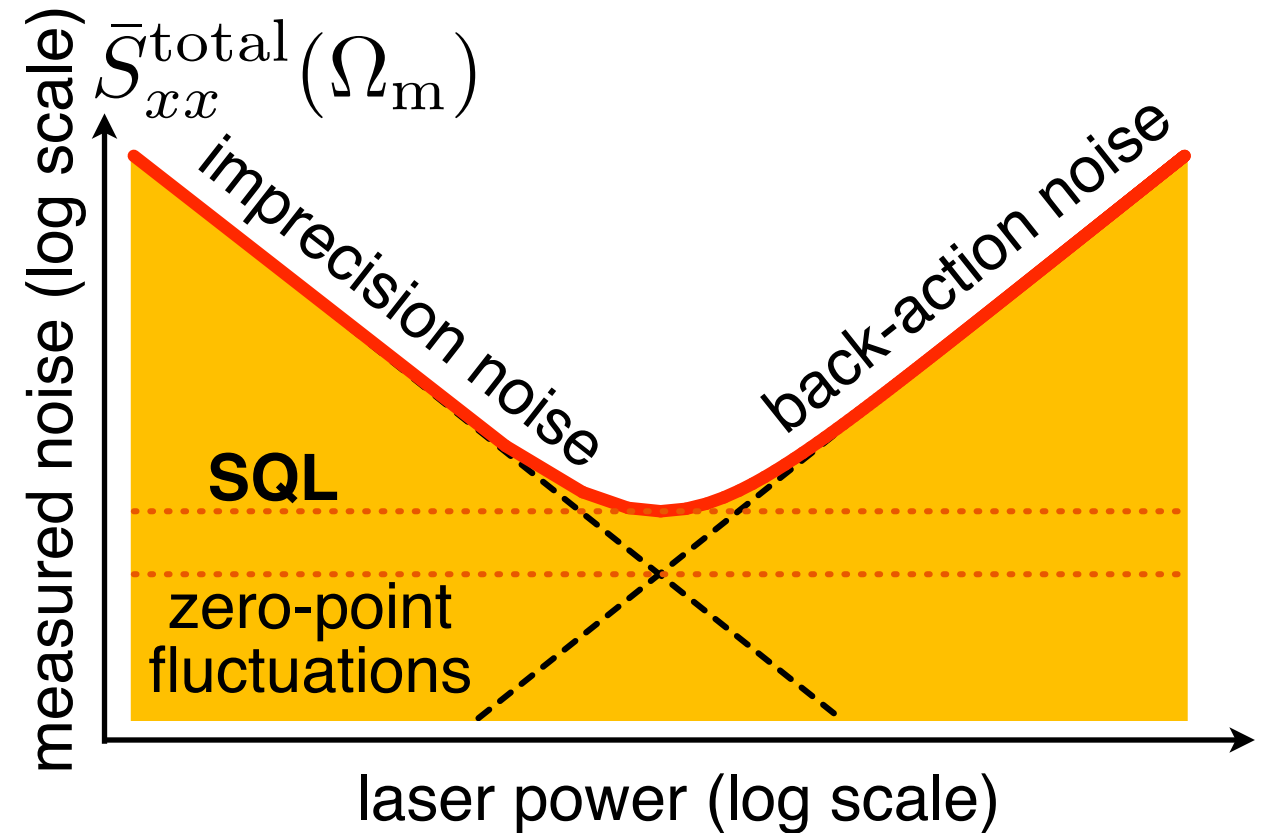
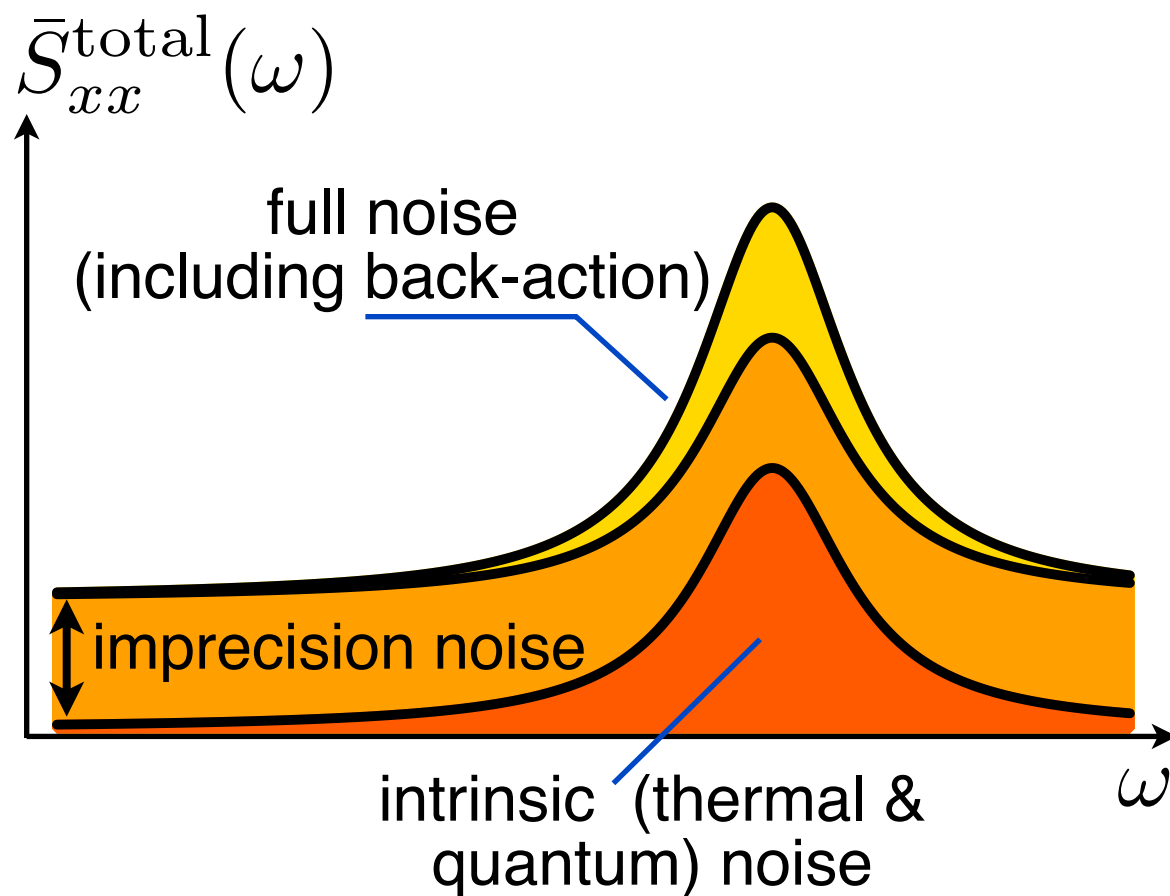
2. measurement back-action:

fluctuating force on system

phase noise of  
laser beam (shot  
noise limit!)

noisy radiation  
pressure force

# “Standard Quantum Limit”



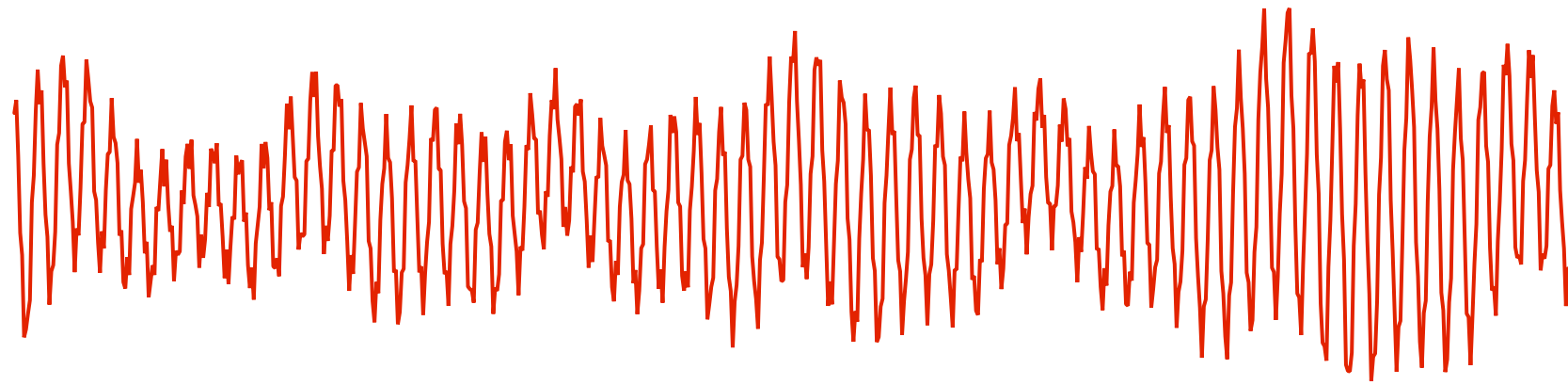
Best case allowed by quantum mechanics:

$$S_{xx}^{(\text{meas})}(\omega) \geq 2 \cdot S_{xx}^{T=0}(\omega)$$

“Standard quantum limit (SQL) of displacement detection”

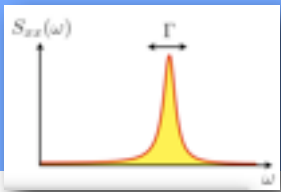
...as if adding the zero-point fluctuations a second time: “adding half a photon”

# Notes on the SQL



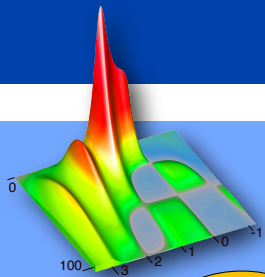
- “**weak measurement**”: integrating the signal over time to suppress the noise
- trying to detect slowly varying “quadratures of motion”:  $\hat{x}(t) = \hat{X}_1 \cos(\omega_M t) + \hat{X}_2 \sin(\omega_M t)$   
 $[\hat{X}_1, \hat{X}_2] = 2x_{\text{ZPF}}^2$  **Heisenberg is the reason for SQL!**  
**no limit for instantaneous measurement of  $x(t)$ !**
- SQL means: detect  $\hat{X}_{1,2}$  down to  $x_{\text{ZPF}}$  on a time scale  $1/\Gamma$  **Impressive:**  $x_{\text{ZPF}} \sim 10^{-15} m$  !

# Optomechanics (Outline)

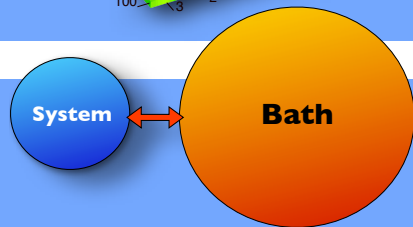


Displacement detection

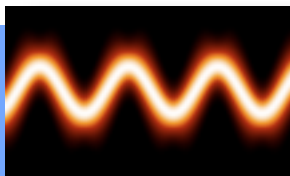
Linear optomechanics



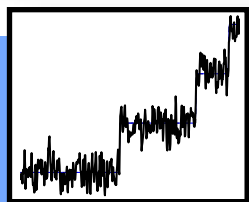
Nonlinear dynamics



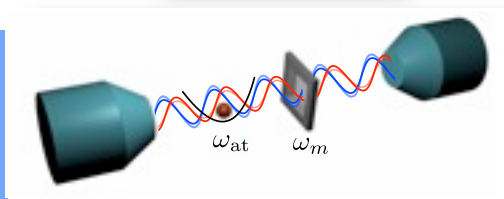
Quantum theory of cooling



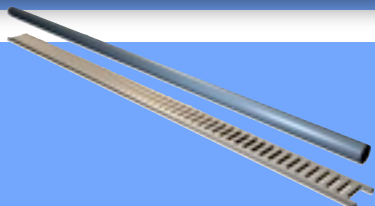
Interesting quantum states



Towards Fock state detection



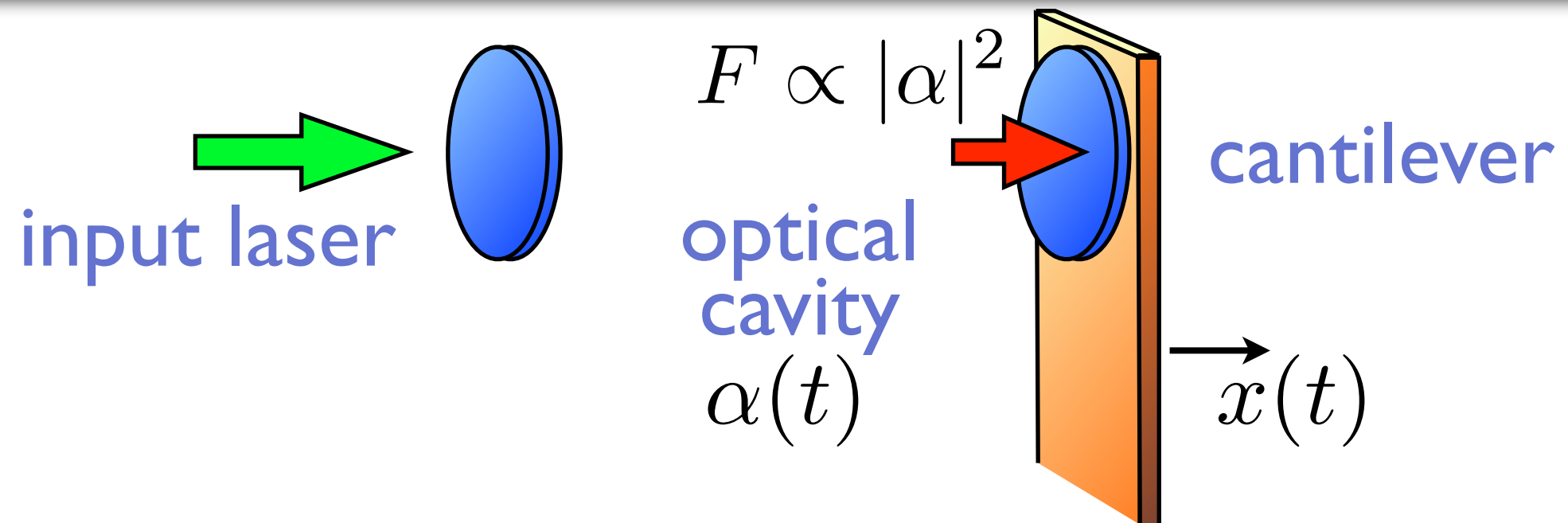
Hybrid systems: coupling to atoms



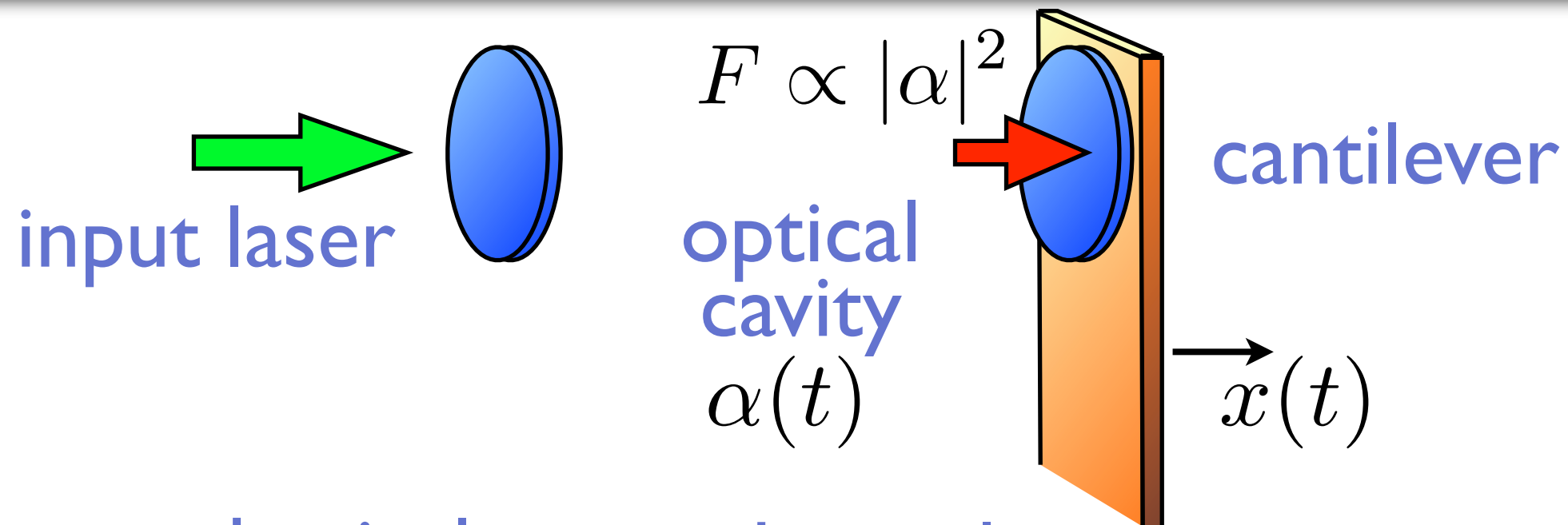
Optomechanical crystals & arrays



# Equations of motion



# Equations of motion



$$\ddot{x} = -\omega_M^2 (x - x_0) - \Gamma \dot{x} + F/m$$

mechanical frequency

mechanical damping

equilibrium position

radiation pressure

$$F = \frac{\hbar \omega_R}{L} |\alpha|^2$$

$$\dot{\alpha} = i\omega_R \frac{x}{L} \alpha - \frac{\kappa}{2} (\alpha - \alpha_{\text{in}})$$

detuning from resonance

cavity decay rate

laser amplitude

# Linearized optomechanics

$$\alpha(t) = \bar{\alpha} + \delta\alpha(t)$$

$$x(t) = \bar{x} + \delta x(t)$$

$\Rightarrow \dots \Rightarrow$

(solve for arbitrary  $F_{\text{ext}}(\omega)$  )

$$\delta x(\omega) = \frac{1}{\underbrace{m(\omega_M^2 - \omega^2) - im\omega\Gamma + \Sigma(\omega)}_{\chi_{xx}^{\text{eff}}(\omega)}} F_{\text{ext}}(\omega)$$

$$\delta\omega_M^2 = \frac{1}{m} \text{Re}\Sigma(\omega_M)$$

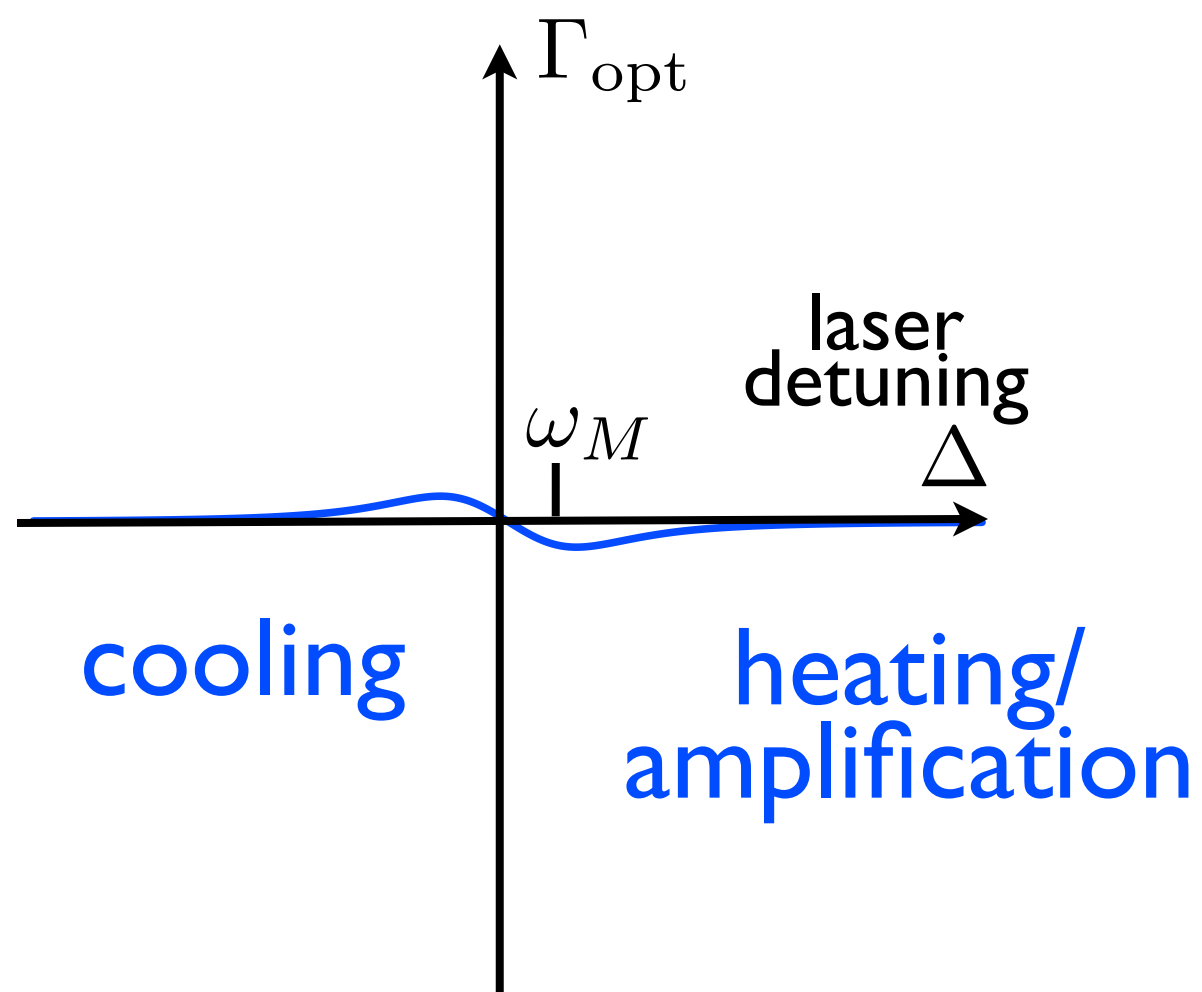
$$\Gamma_{\text{opt}} = -\frac{1}{m\omega_M} \text{Im}\Sigma(\omega_M)$$

Optomechanical  
frequency shift  
("optical spring")

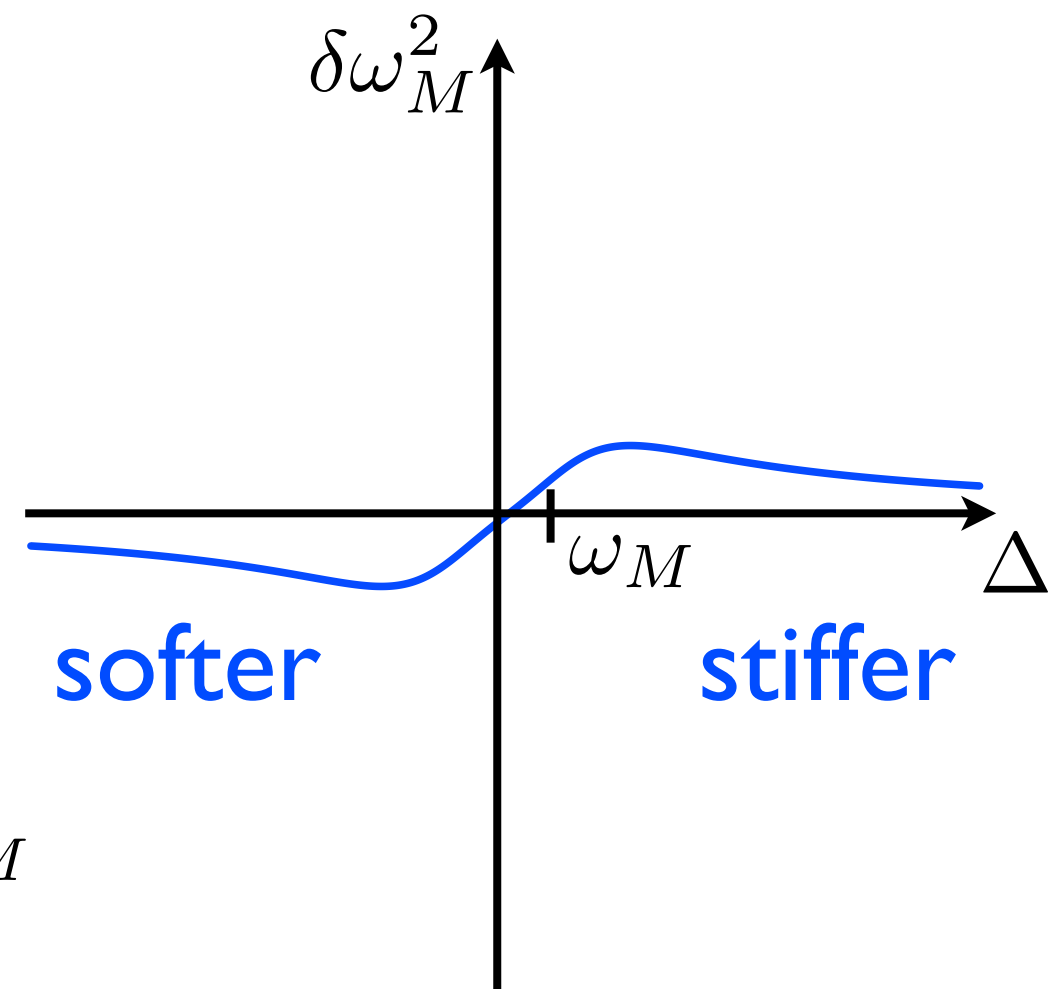
Effective  
optomechanical  
damping rate

# Linearized dynamics

**Effective  
optomechanical  
damping rate**



**Optomechanical  
frequency shift  
("optical spring")**



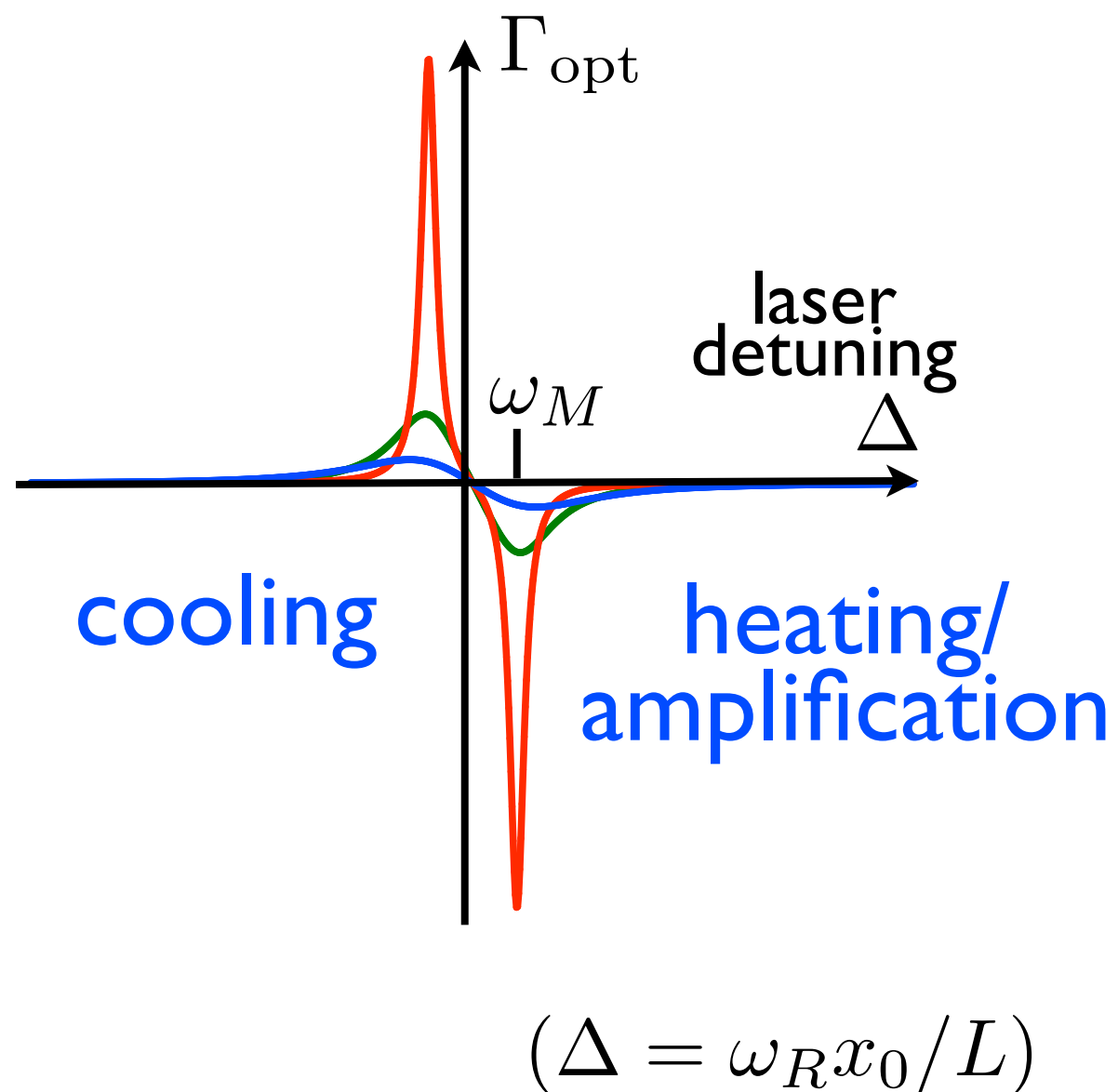
$\frac{\kappa}{\omega_M}$   
■ 2

$(\Delta = \omega_R x_0 / L)$

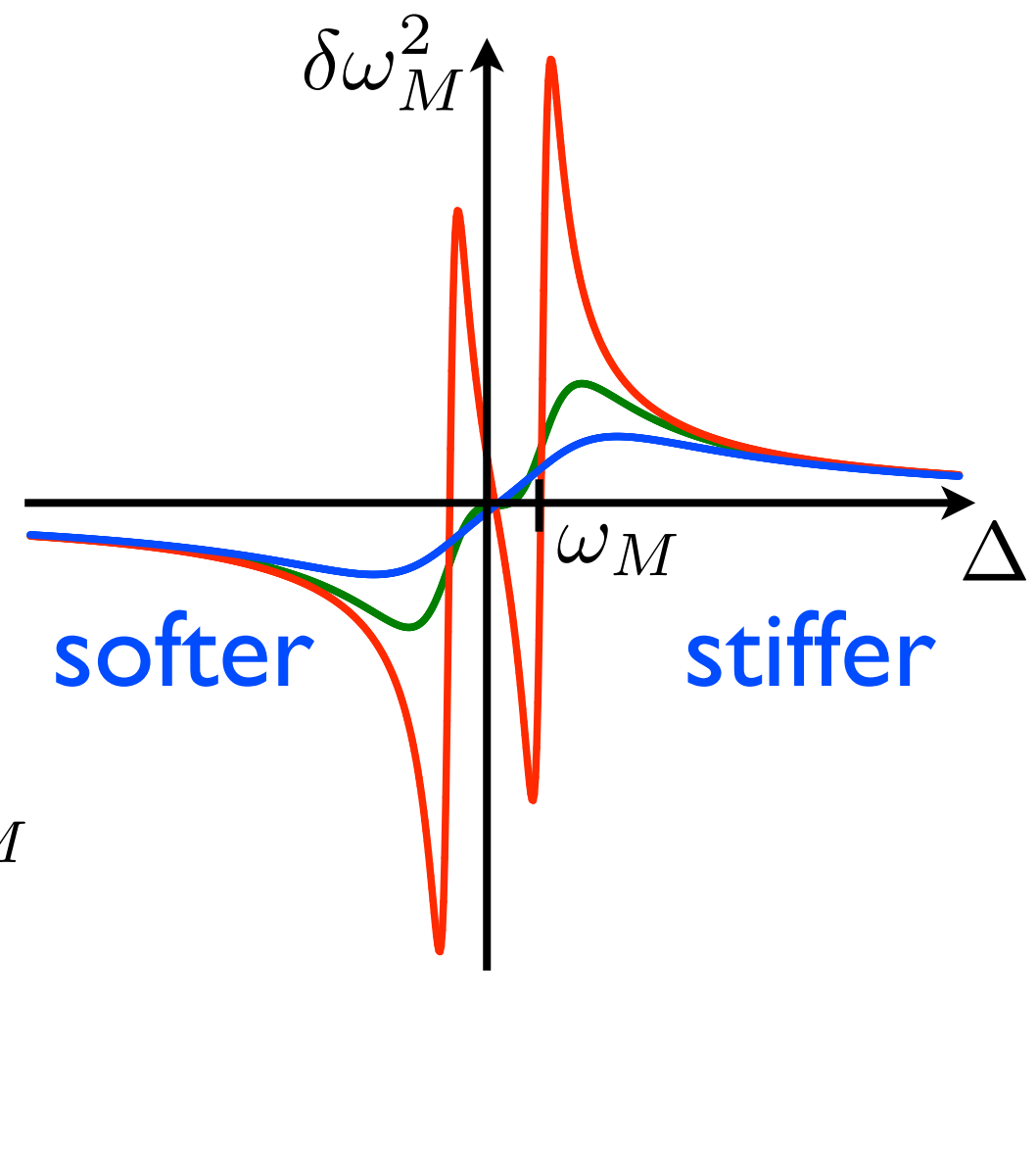


# Linearized dynamics

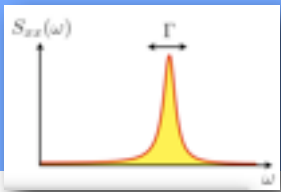
Effective  
optomechanical  
damping rate



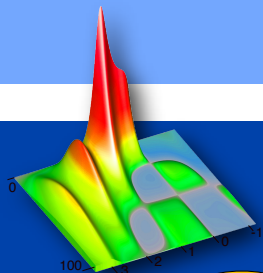
Optomechanical  
frequency shift  
("optical spring")



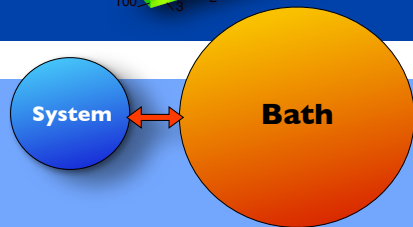
# Optomechanics (Outline)



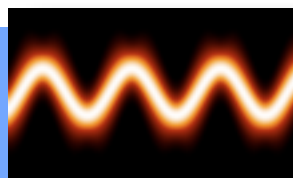
Displacement detection



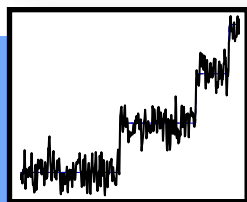
Linear optomechanics



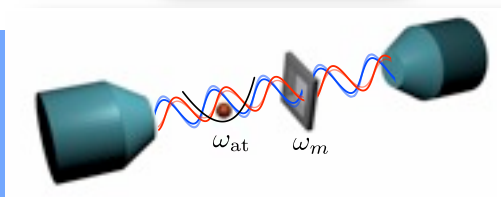
Nonlinear dynamics



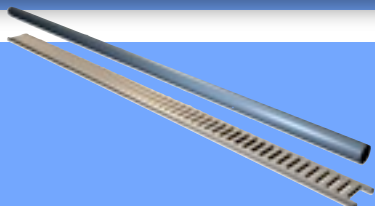
Interesting quantum states



Towards Fock state detection

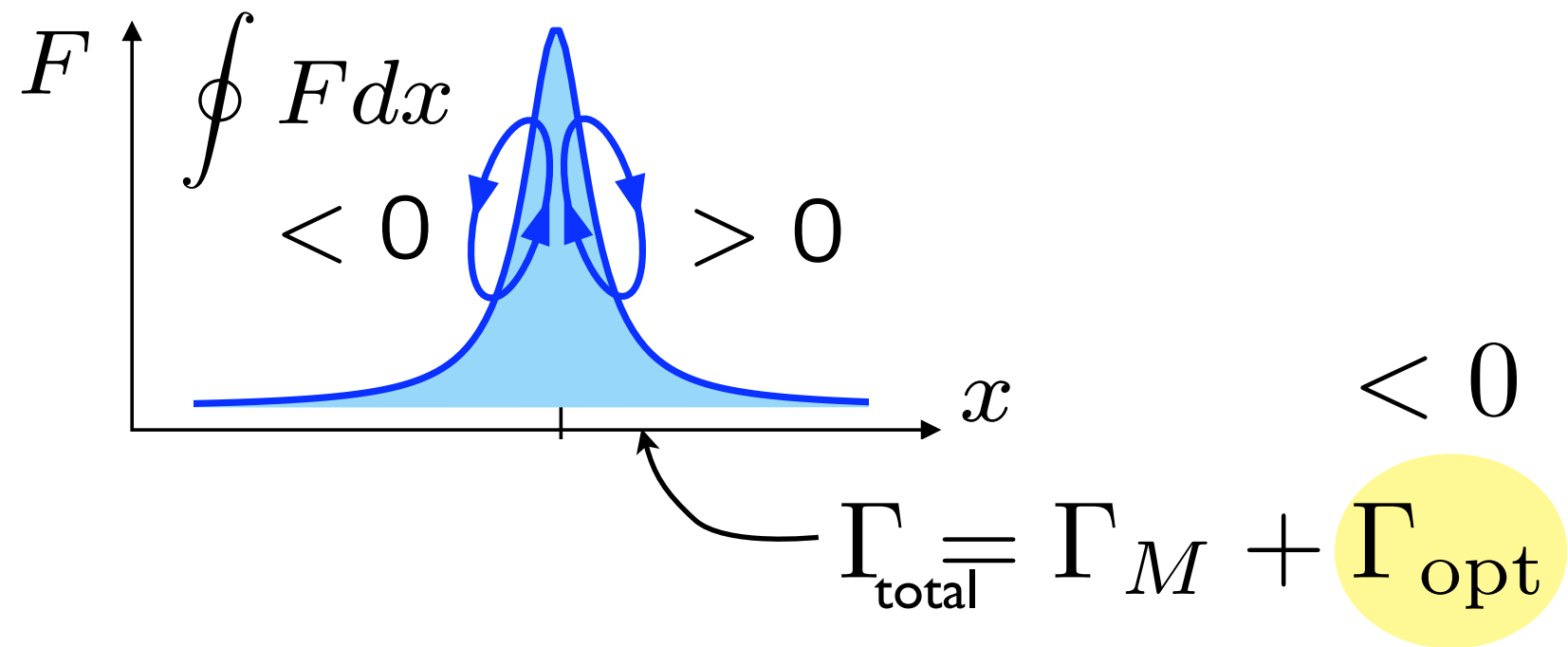


Hybrid systems: coupling to atoms

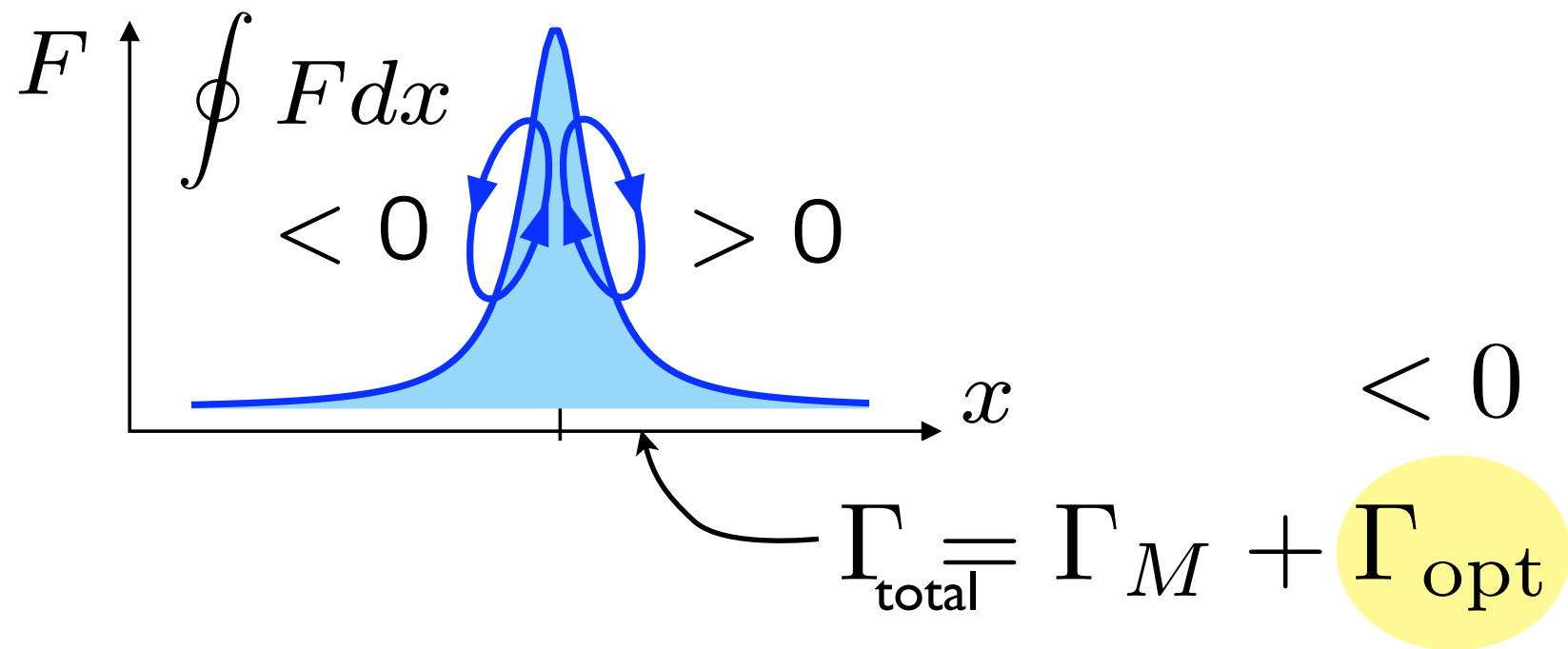


Optomechanical crystals & arrays

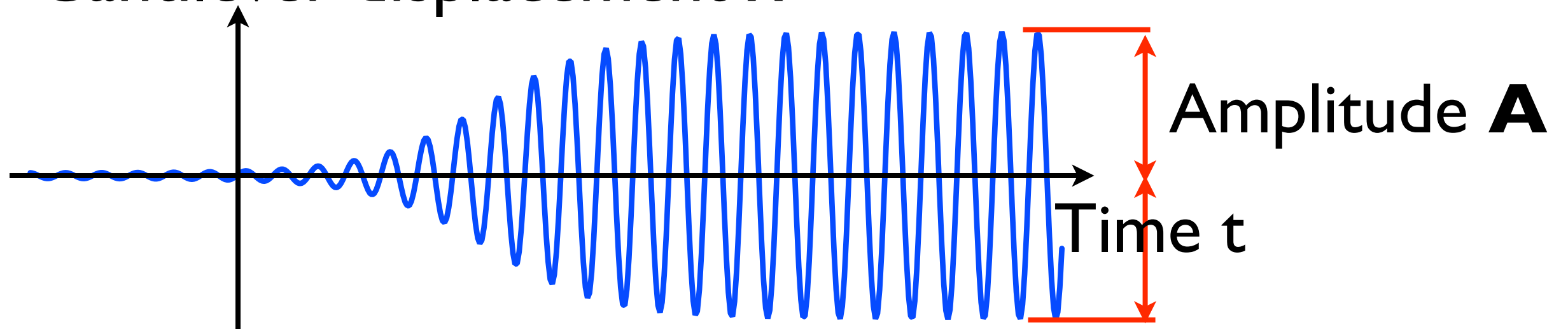
# Self-induced oscillations



# Self-induced oscillations

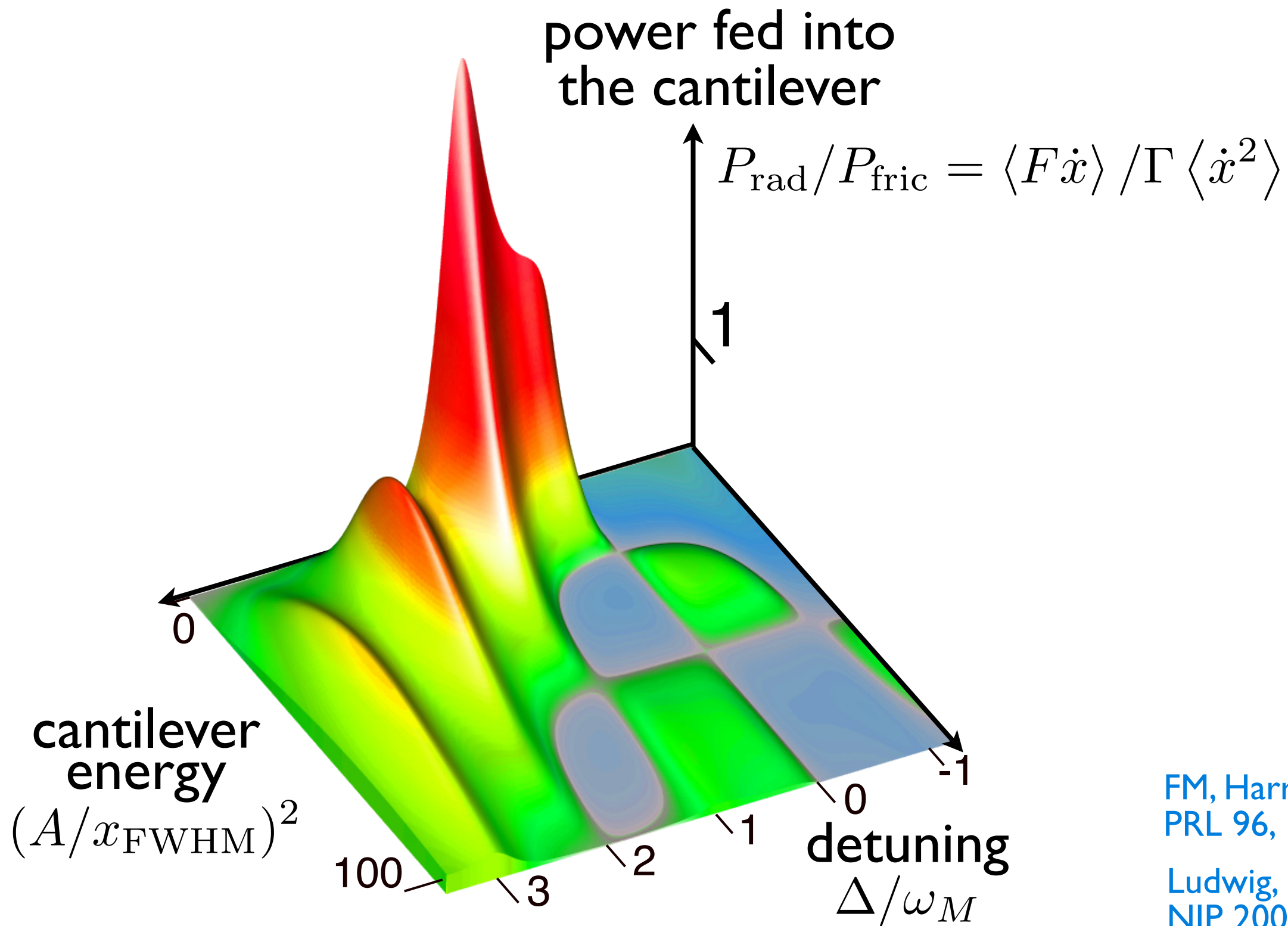


Beyond some laser input power threshold: instability  
Cantilever displacement  $x$





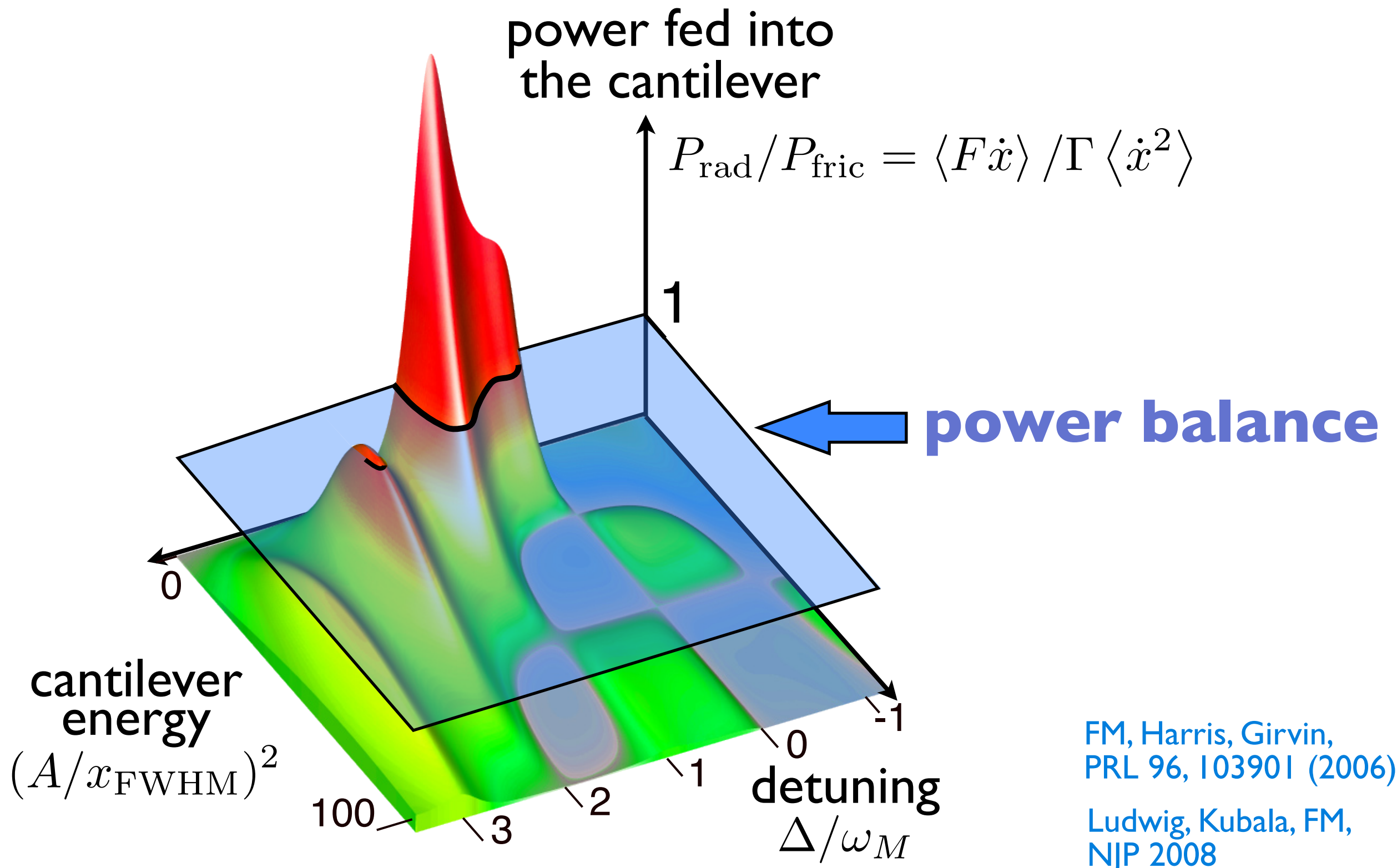
# Attractor diagram



FM, Harris, Girvin,  
PRL 96, 103901 (2006)

Ludwig, Kubala, FM,  
NJP 2008

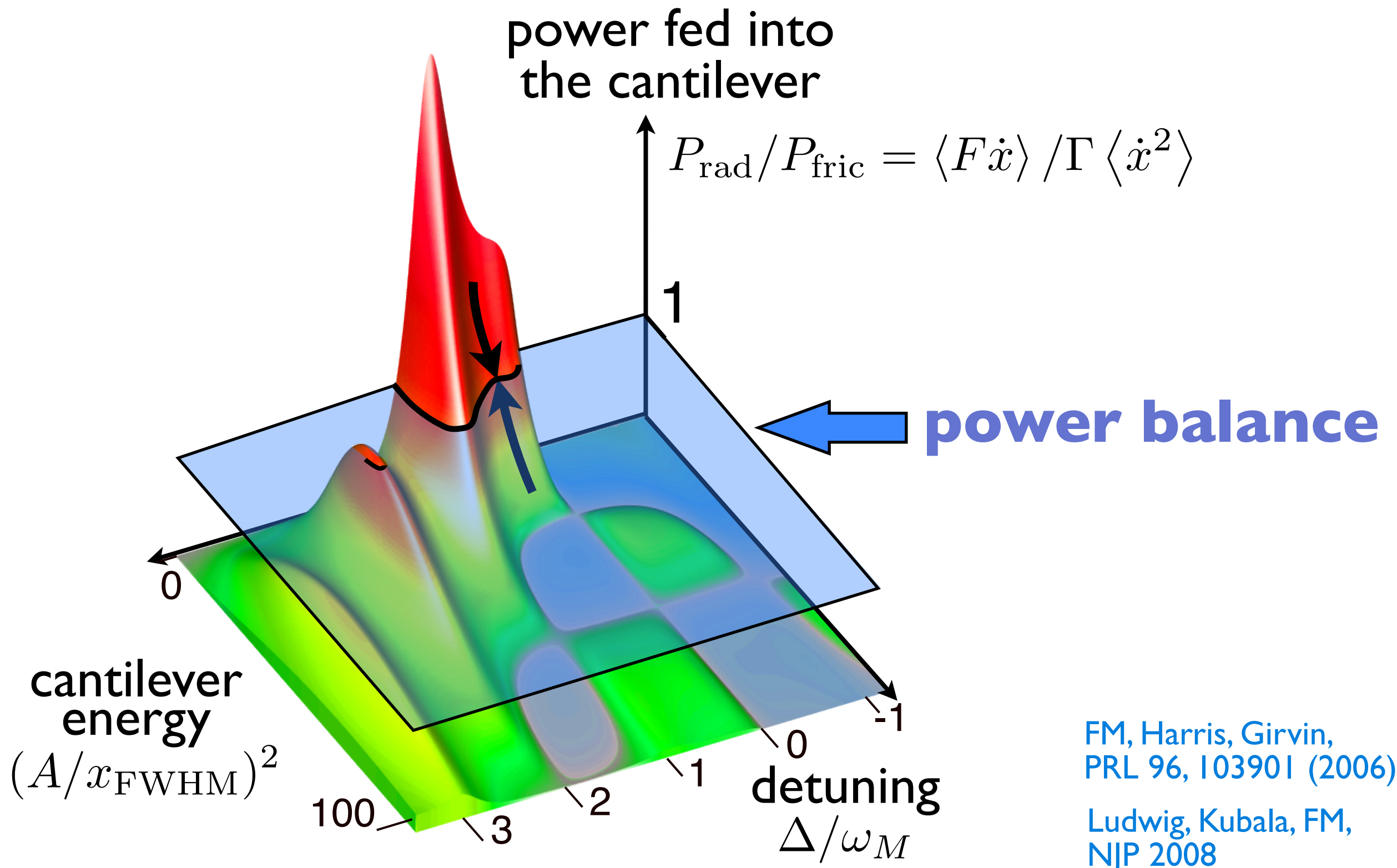
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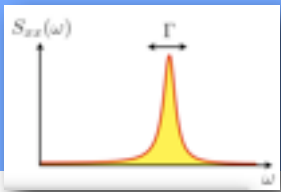
# Attractor diagram



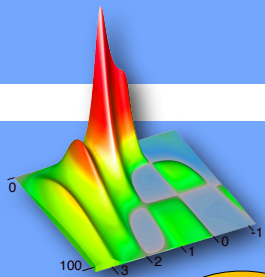
FM, Harris, Girvin,  
PRL 96, 103901 (2006)

Ludwig, Kubala, FM,  
NJP 2008

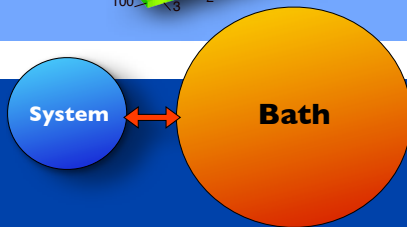
# Optomechanics (Outline)



Displacement detection

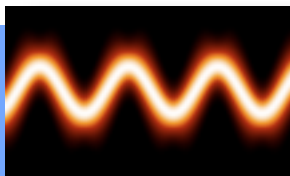


Linear optomechanics

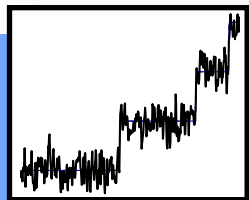


Nonlinear dynamics

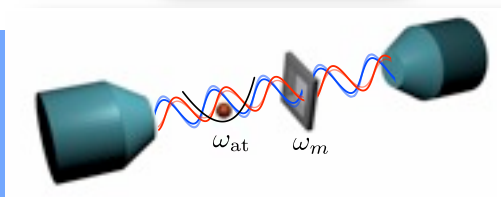
Quantum theory of cooling



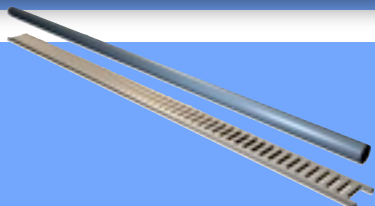
Interesting quantum states



Towards Fock state detection

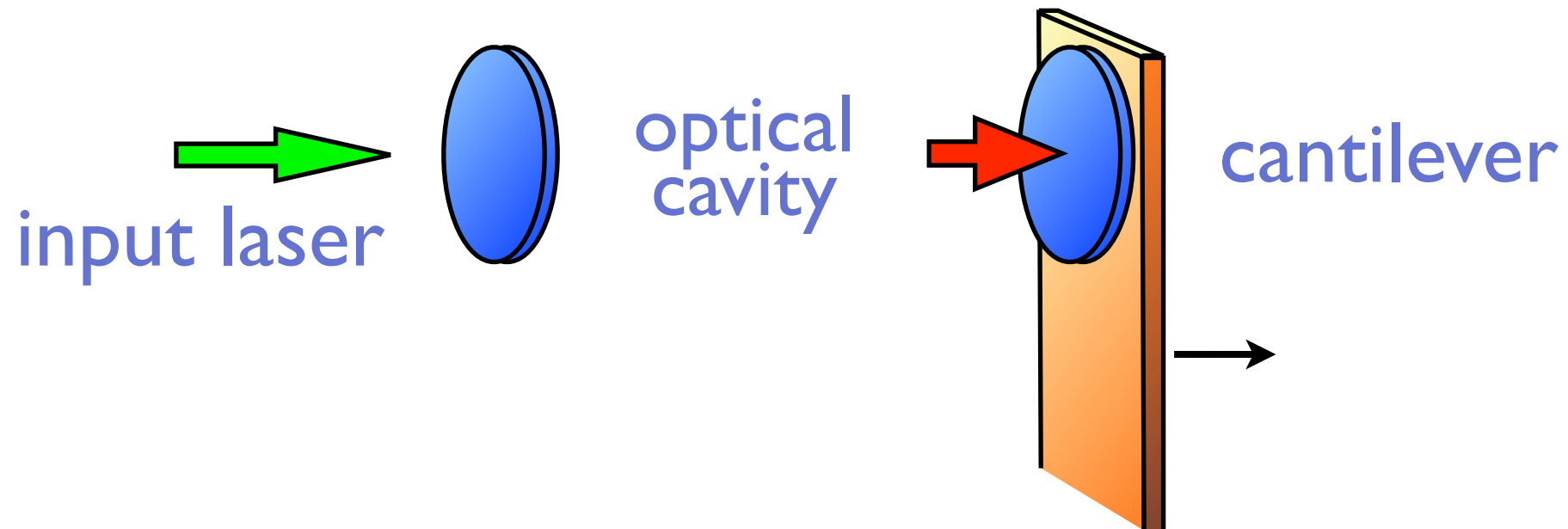


Hybrid systems: coupling to atoms



Optomechanical crystals & arrays

# Cooling with light



**Current goal in the field: *ground state* of mechanical motion of a macroscopic cantilever**

$$k_B T_{\text{eff}} \ll \hbar \omega_M$$

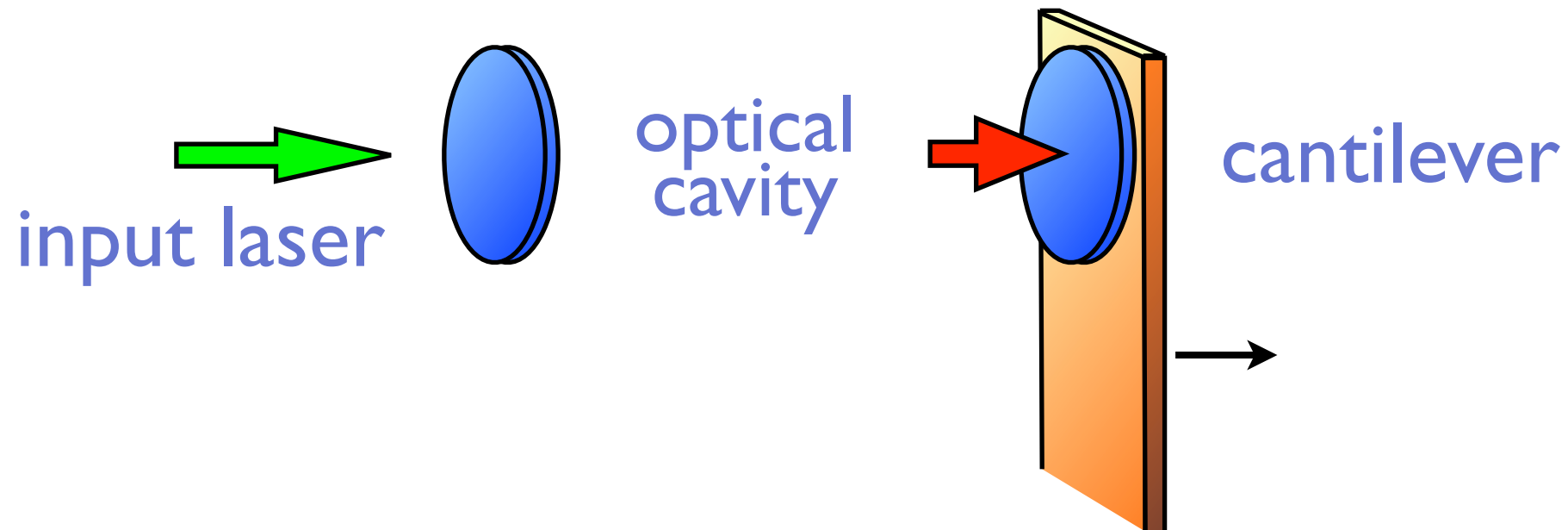
**Classical theory:**

$$T_{\text{eff}} = T \cdot \frac{\Gamma_M}{\Gamma_{\text{opt}} + \Gamma_M}$$

Pioneering theory and experiments: **Braginsky**  
(since 1960s)

optomechanical damping rate

# Cooling with light



**Current goal in the field: *ground state* of mechanical motion of a macroscopic cantilever**

$$k_B T_{\text{eff}} \ll \hbar \omega_M$$

**Classical theory:**

$$T_{\text{eff}} = T \cdot \frac{\Gamma_M}{\Gamma_{\text{opt}} + \Gamma_M} \rightarrow 0 ?$$

**quantum limit?**

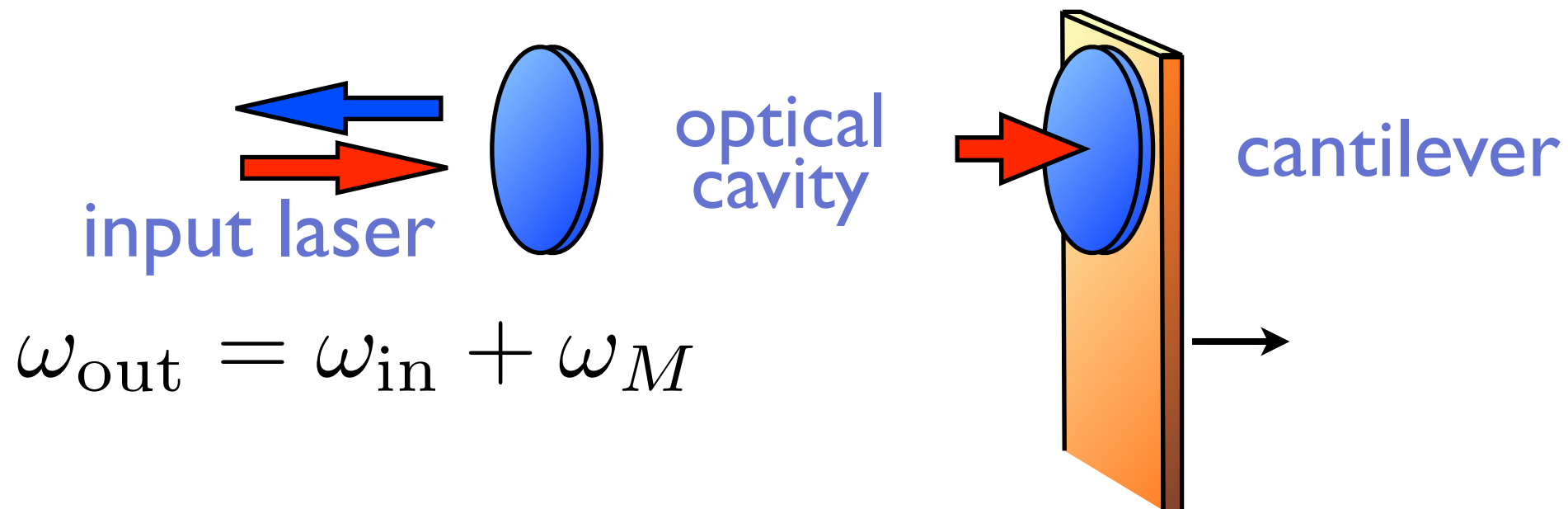
**shot noise!**

Pioneering theory and experiments: **Braginsky**  
(since 1960s)

optomechanical damping rate



# Cooling with light



## Quantum picture: Raman scattering – sideband cooling

### Original idea:

Sideband cooling in ion traps – Hänsch, Schawlow / Wineland, Dehmelt 1975

### Similar ideas proposed for nanomechanics:

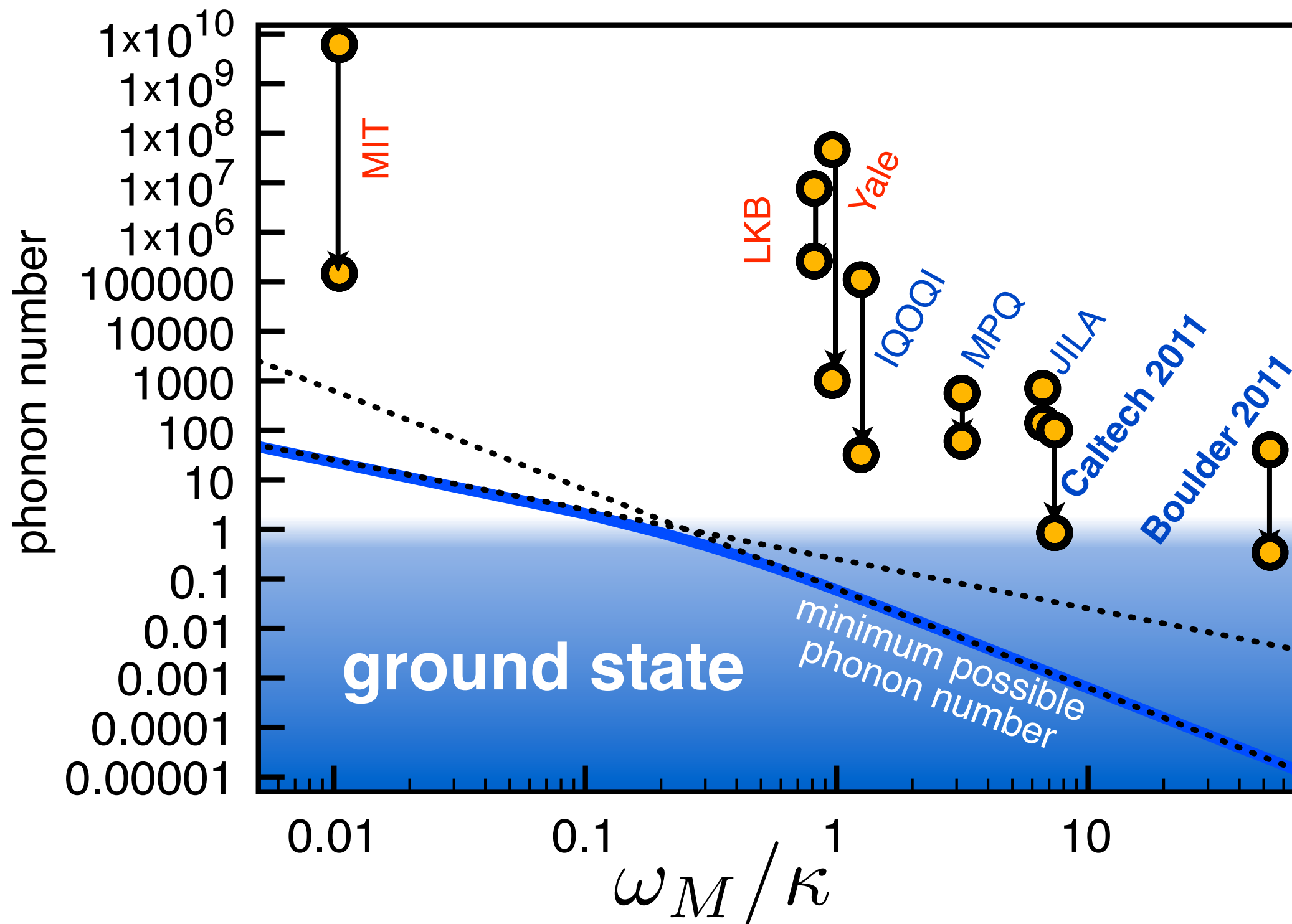
cantilever + quantum dot – Wilson-Rae, Zoller, Imamoglu 2004

cantilever + Cooper-pair box – Martin Shnirman, Tian, Zoller 2004

cantilever + ion – Tian, Zoller 2004

cantilever + supercond. SET – Clerk, Bennett / Blencowe, Imbers, Armour 2005,  
Naik et al. (Schwab group) 2006

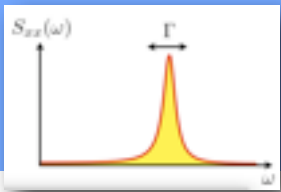
# Laser-cooling towards the ground state



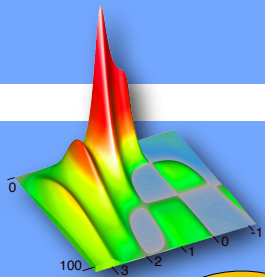
analogy to (cavity-assisted)  
laser cooling of atoms

FM et al., PRL **93**, 093902 (2007)  
Wilson-Rae et al., PRL **99**, 093901 (2007)

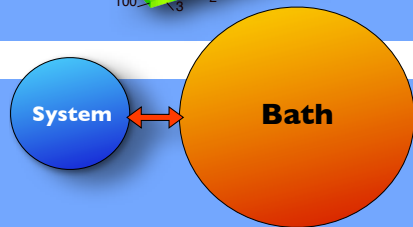
# Optomechanics (Outline)



Displacement detection

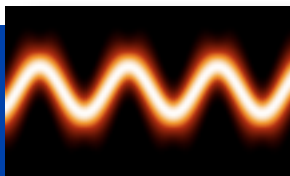


Linear optomechanics

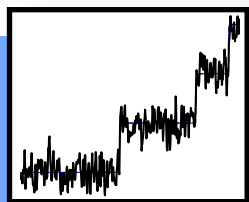


Nonlinear dynamics

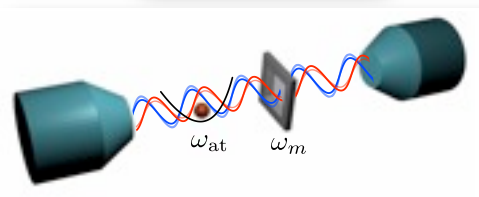
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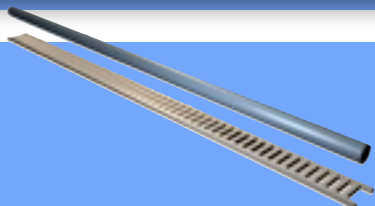
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Towards Fock state detection



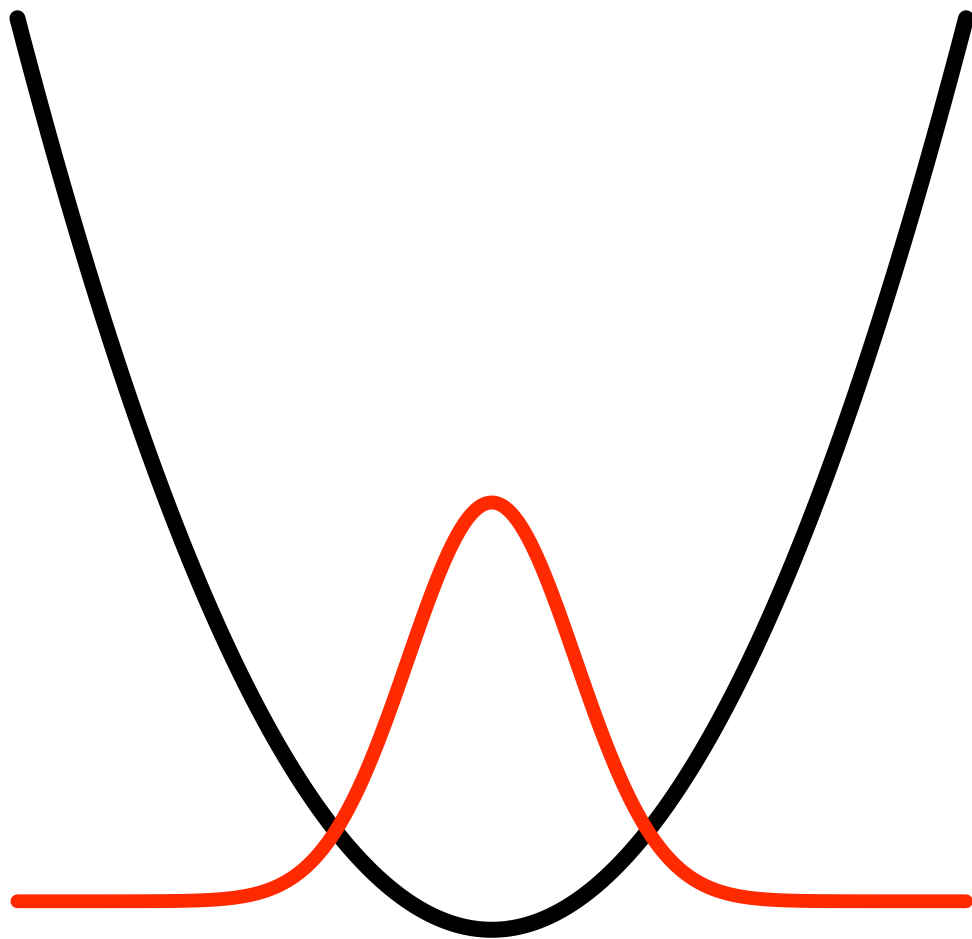
Hybrid systems: coupling to atoms



Optomechanical crystals & arrays

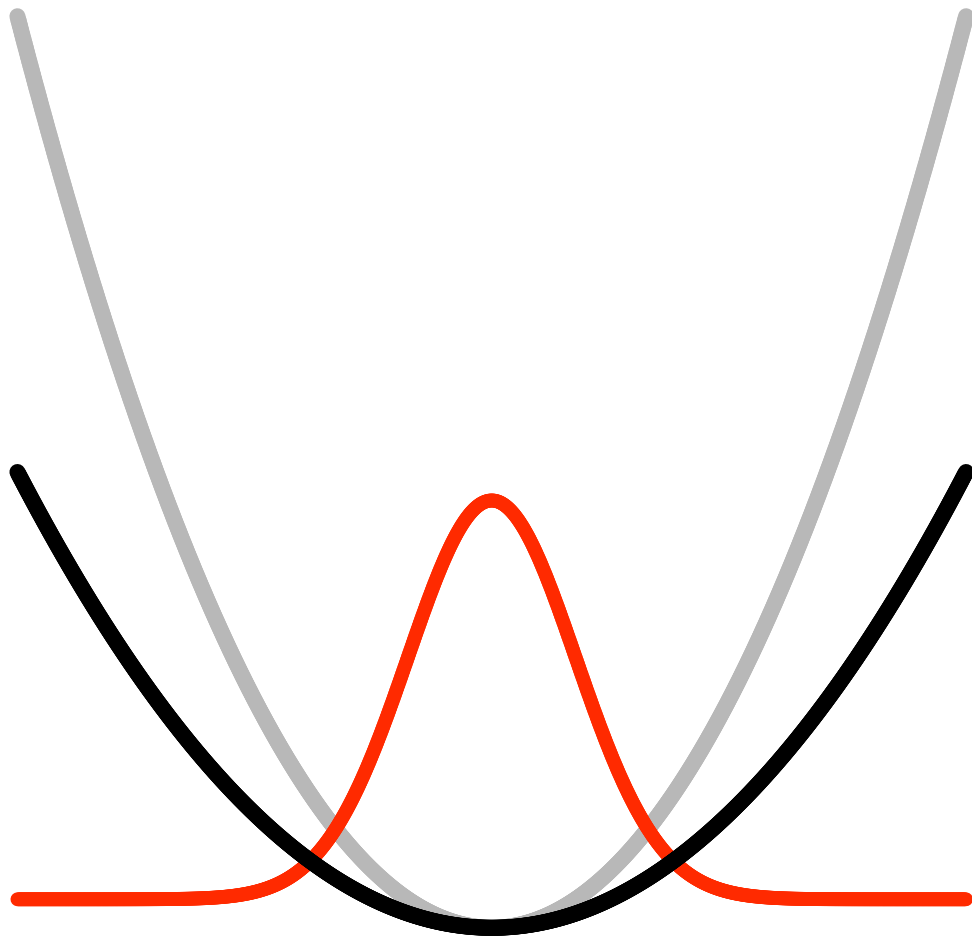
# Squeezed states

Squeezing the mechanical oscillator state



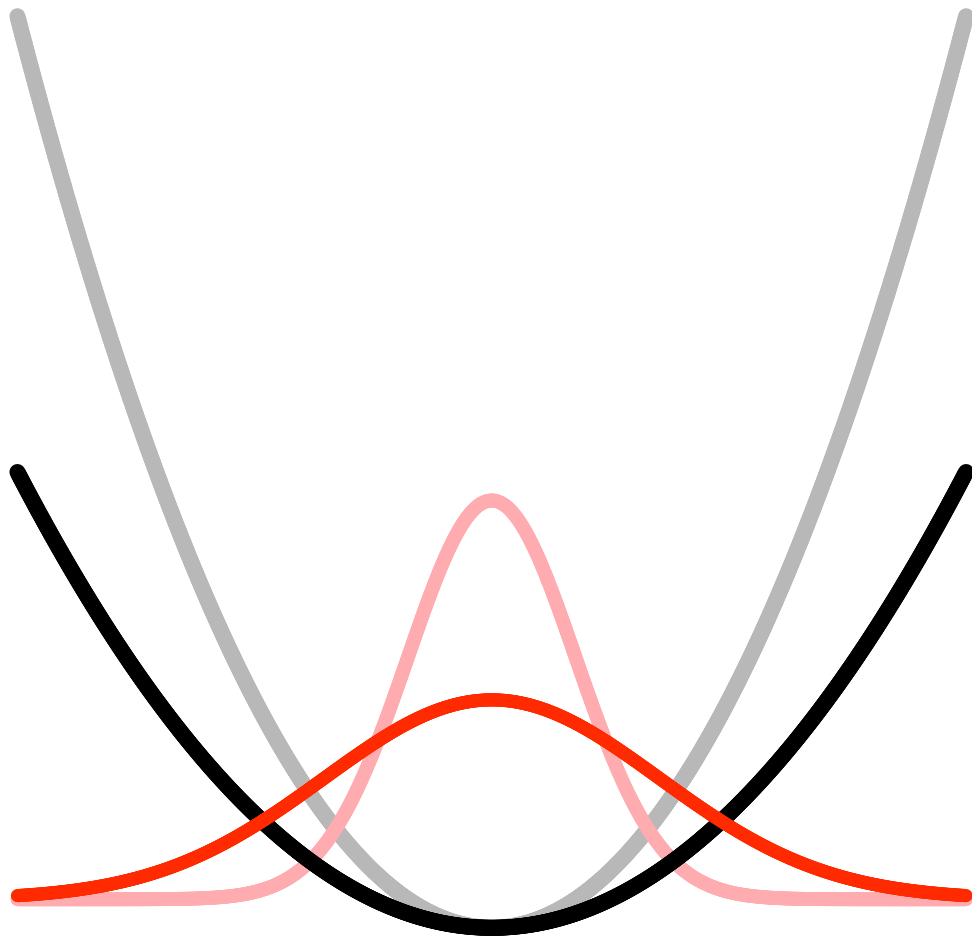
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Squeezing the mechanical oscillator state



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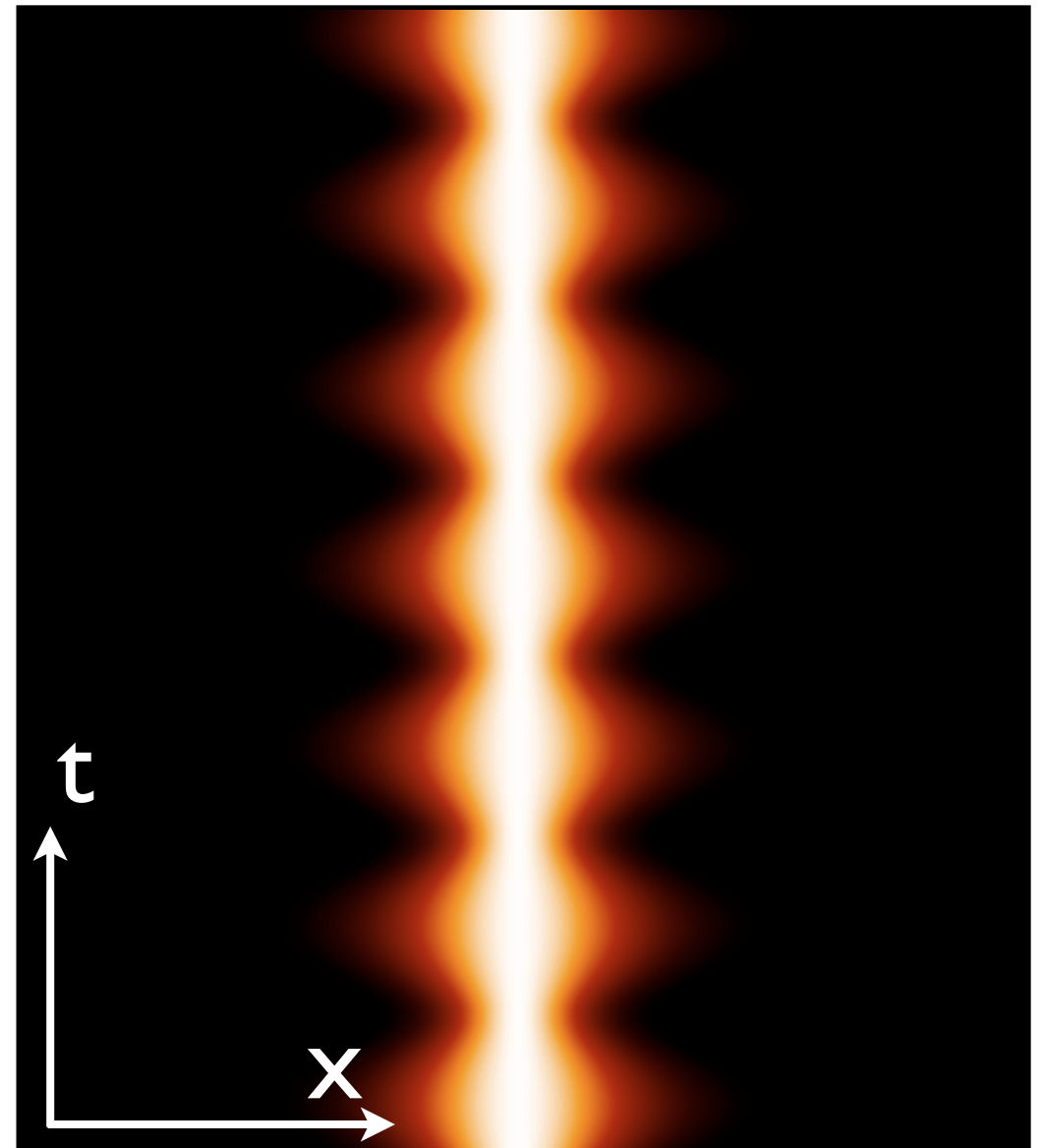
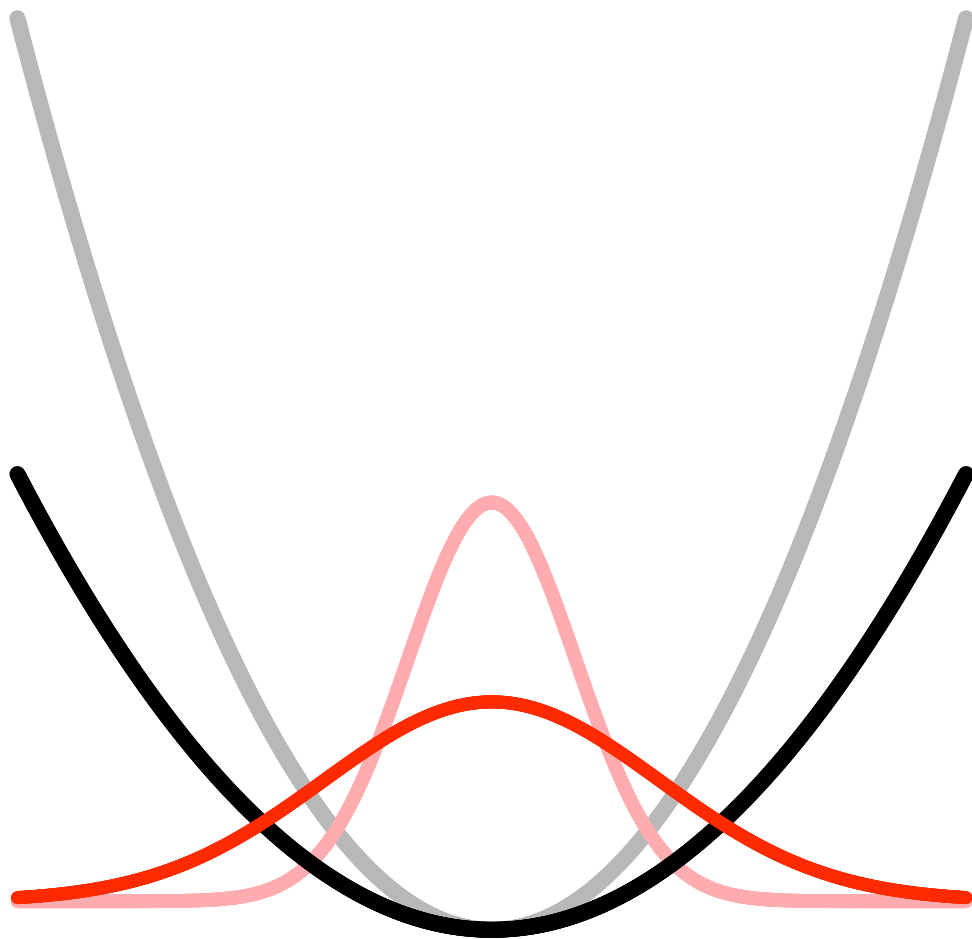
Squeezing the mechanical oscillator state





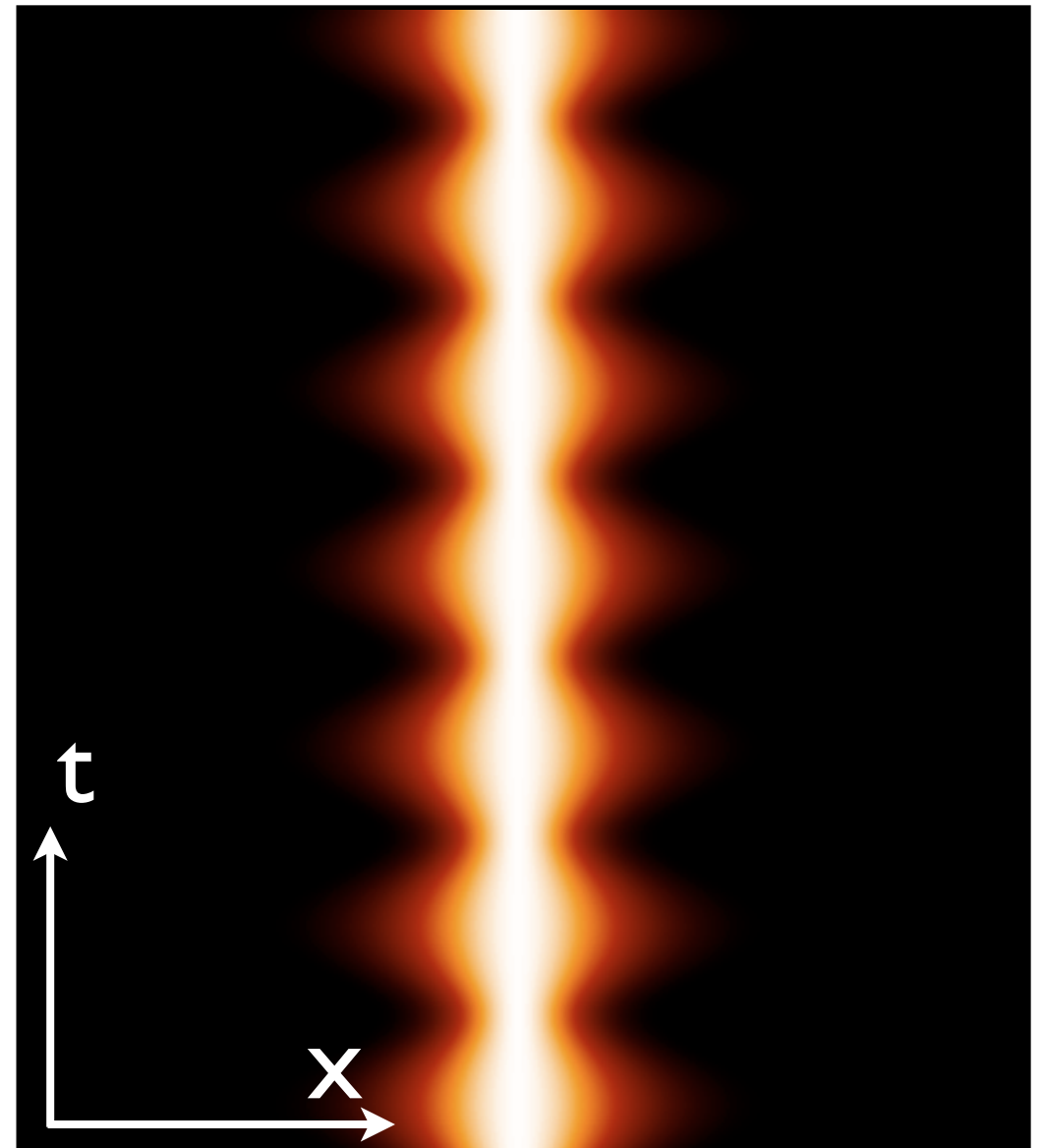
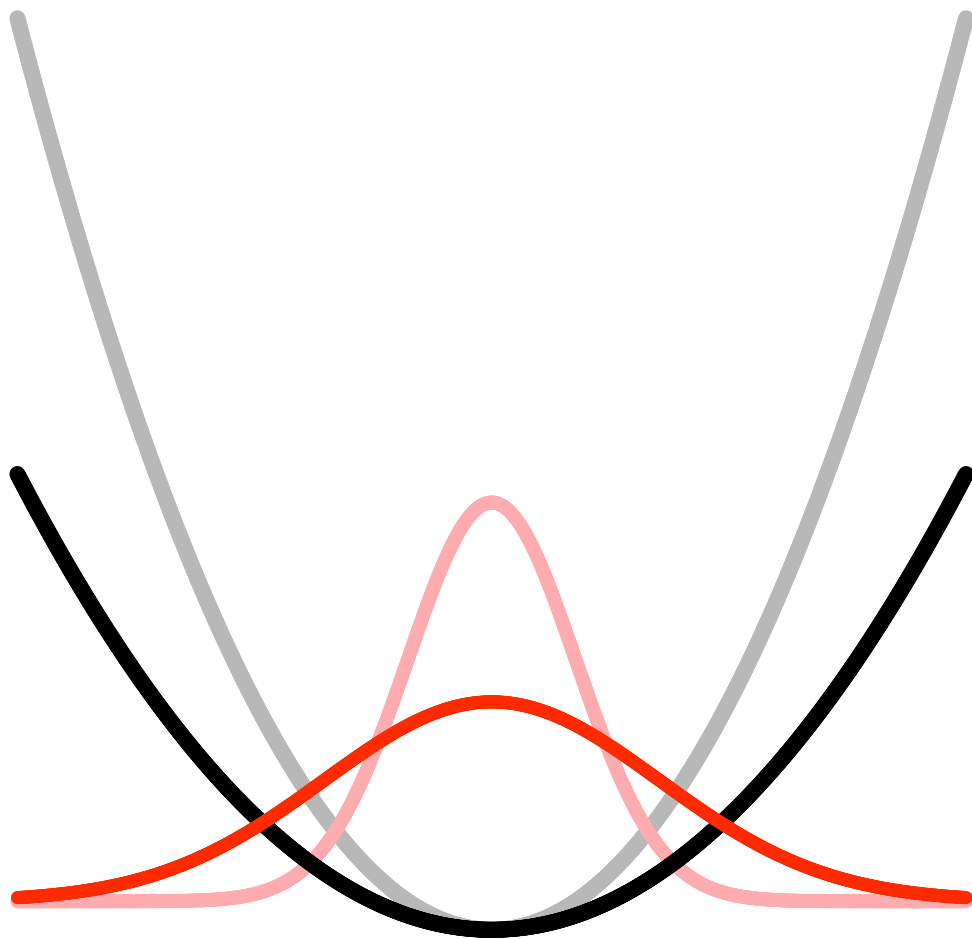
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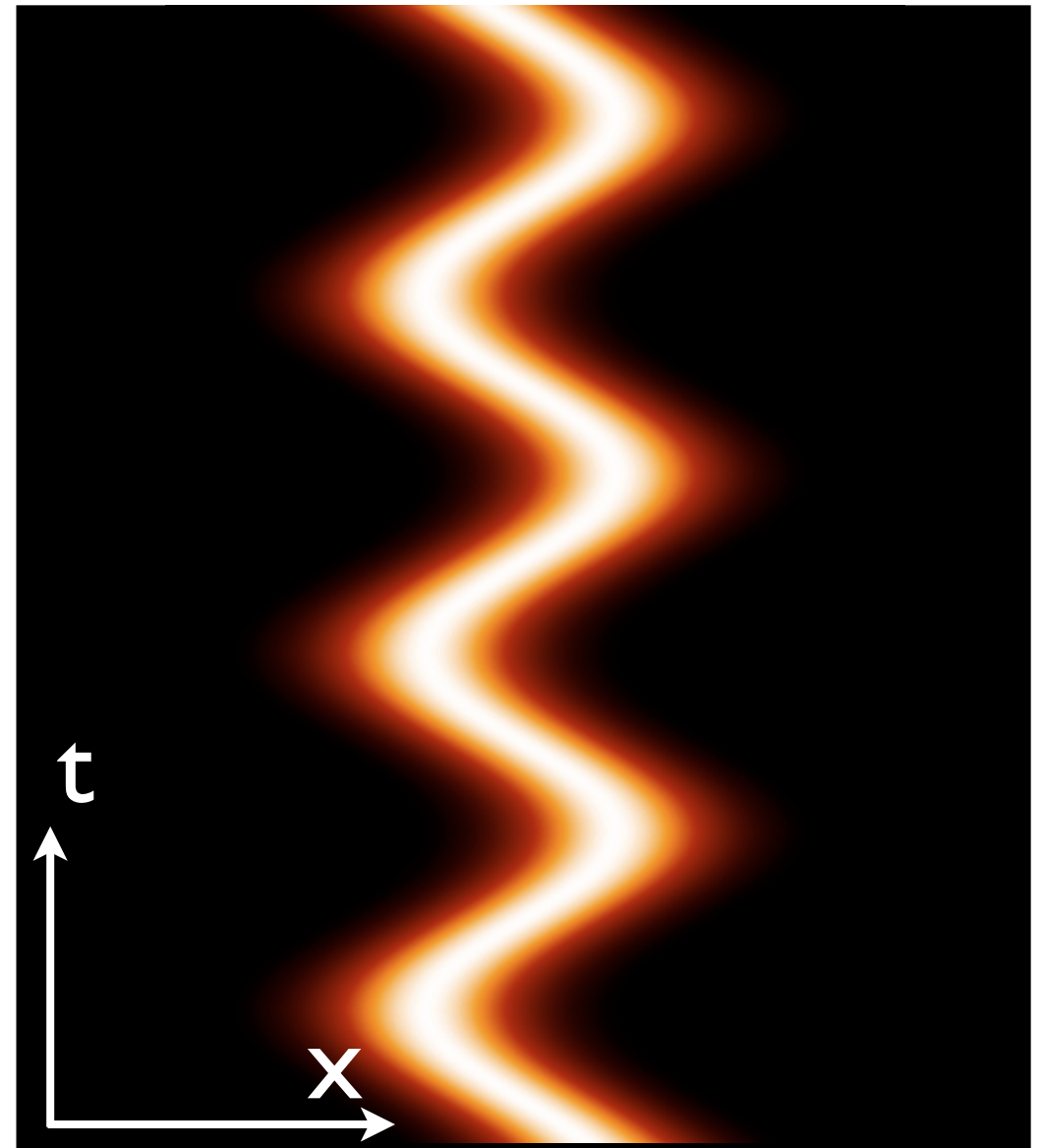
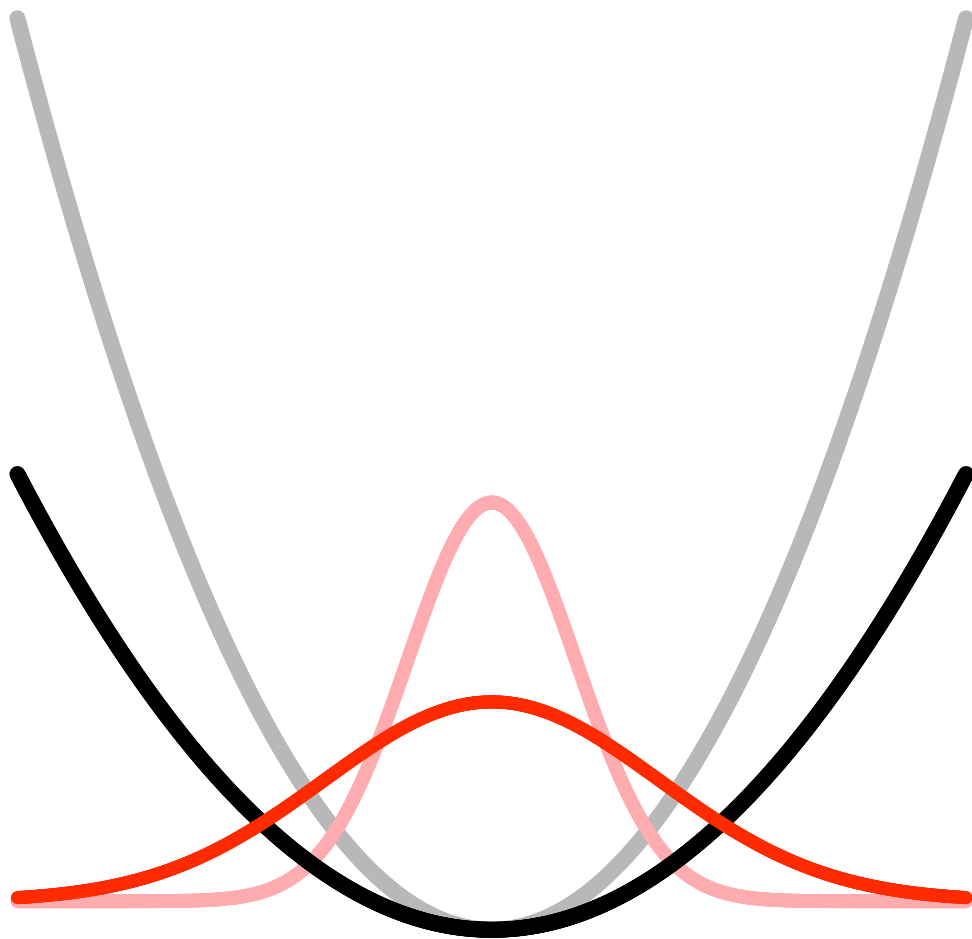
Squeezing the mechanical oscillator state



Periodic modulation of spring constant:  $\delta\omega_M^2(t) \propto \cos(2\omega_M t)$   
Parametric amplification

# Squeezed states

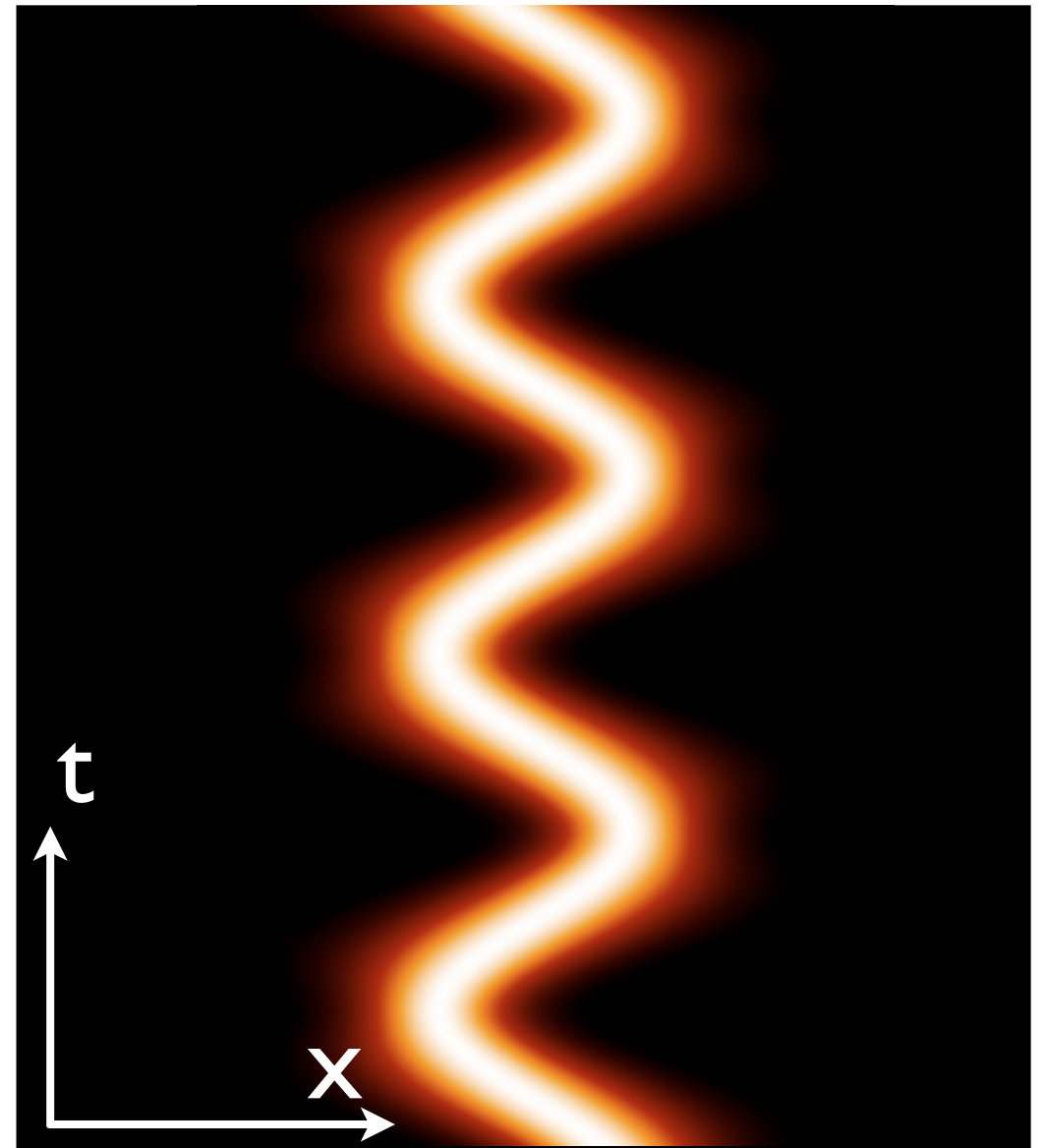
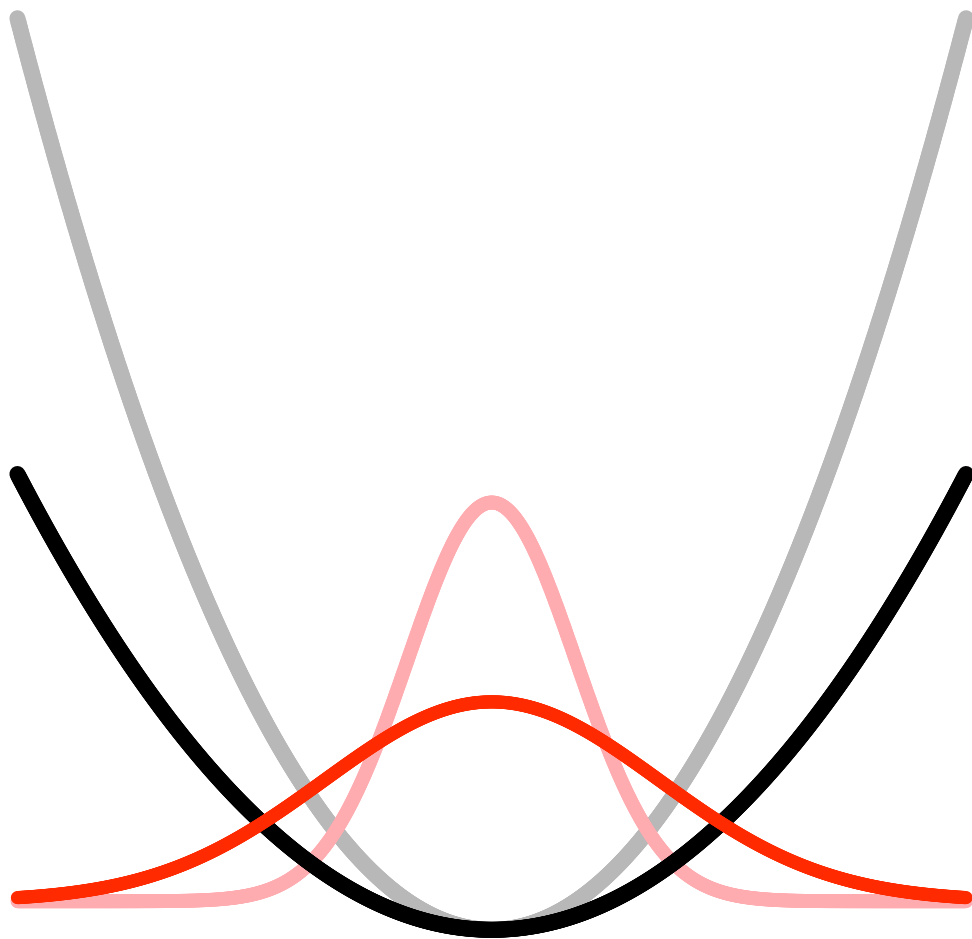
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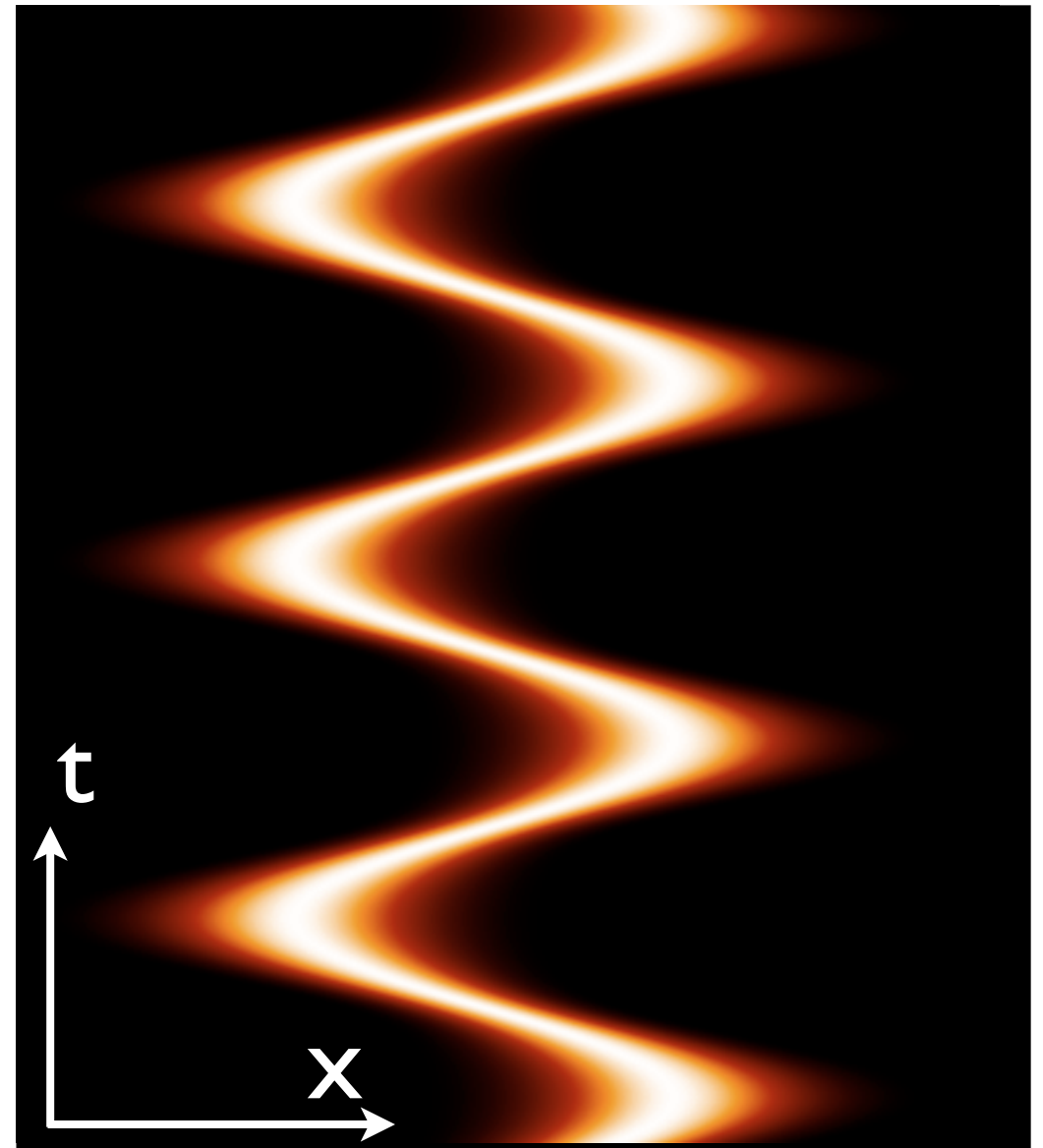
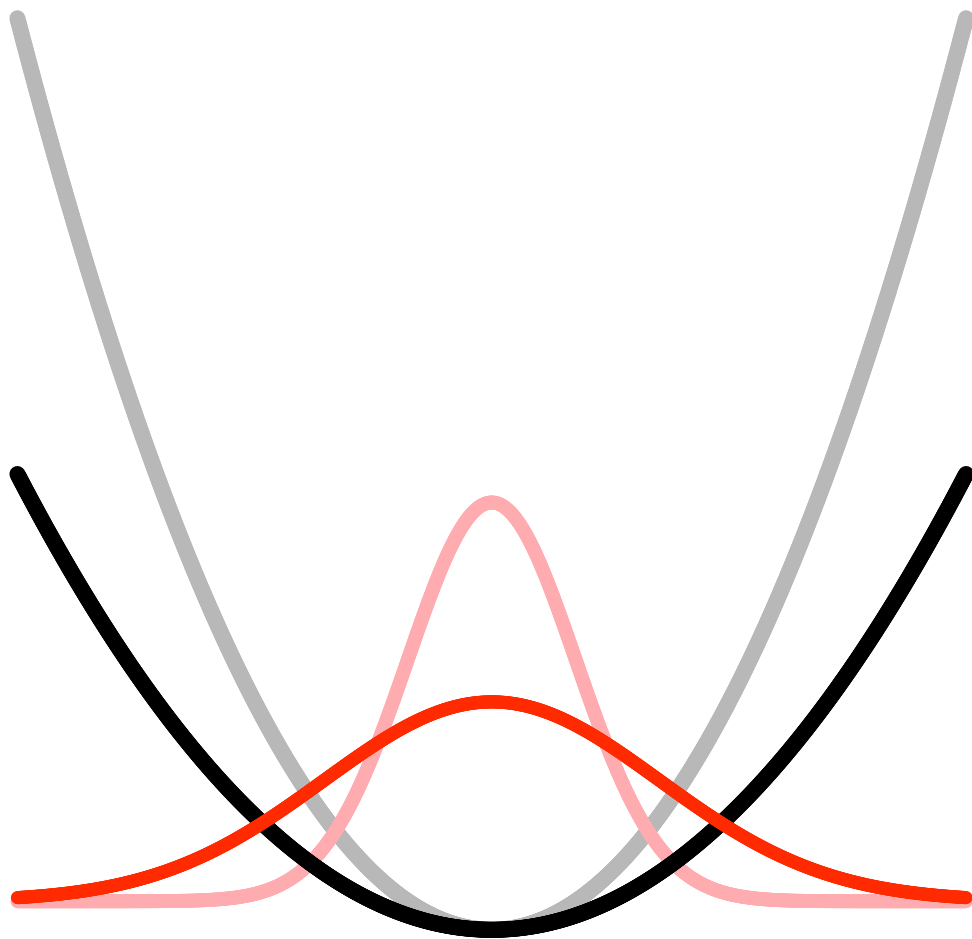
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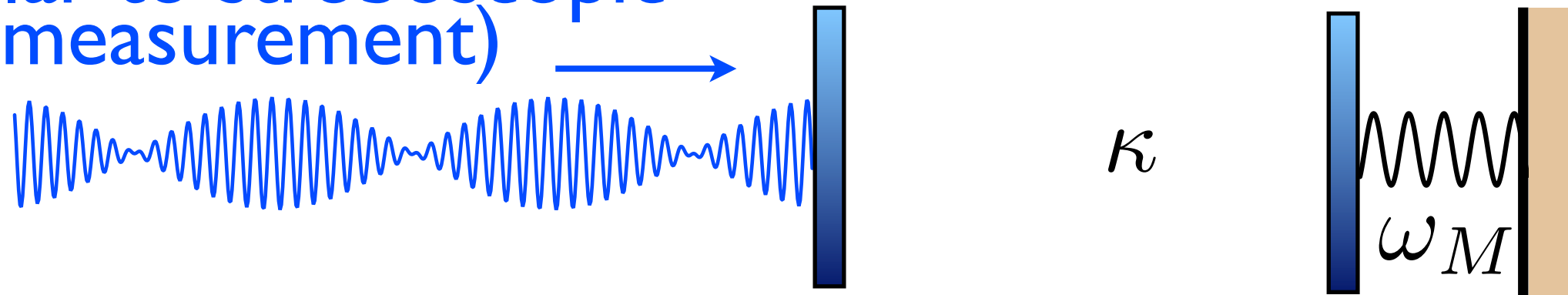


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# Measuring quadratures ("beating the SQL")

Clerk, Marquardt, Jacobs; NJP **10**, 095010 (2008)

amplitude-modulated input field  
(similar to stroboscopic  
measurement)



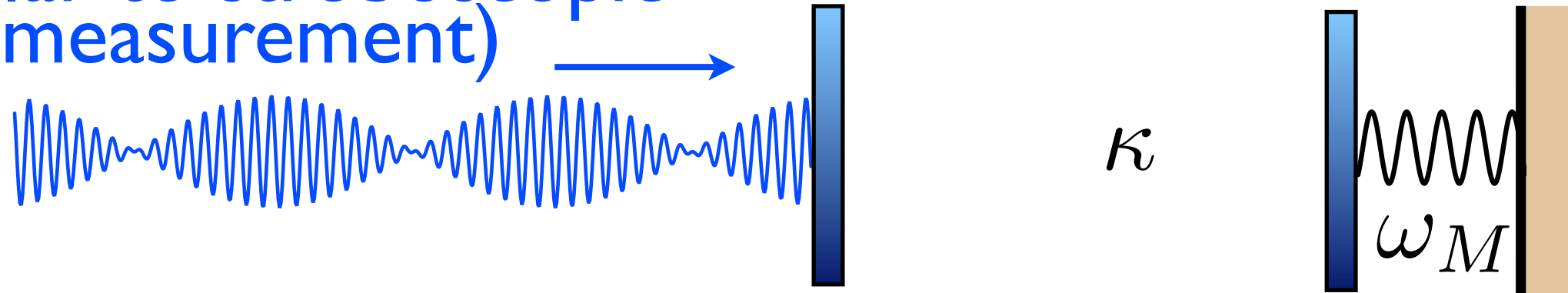
measure only one quadrature, back-action noise affects  
only the other one....need:  $\kappa \ll \omega_M$



# Measuring quadratures ("beating the SQL")

Clerk, Marquardt, Jacobs; NJP **10**, 095010 (2008)

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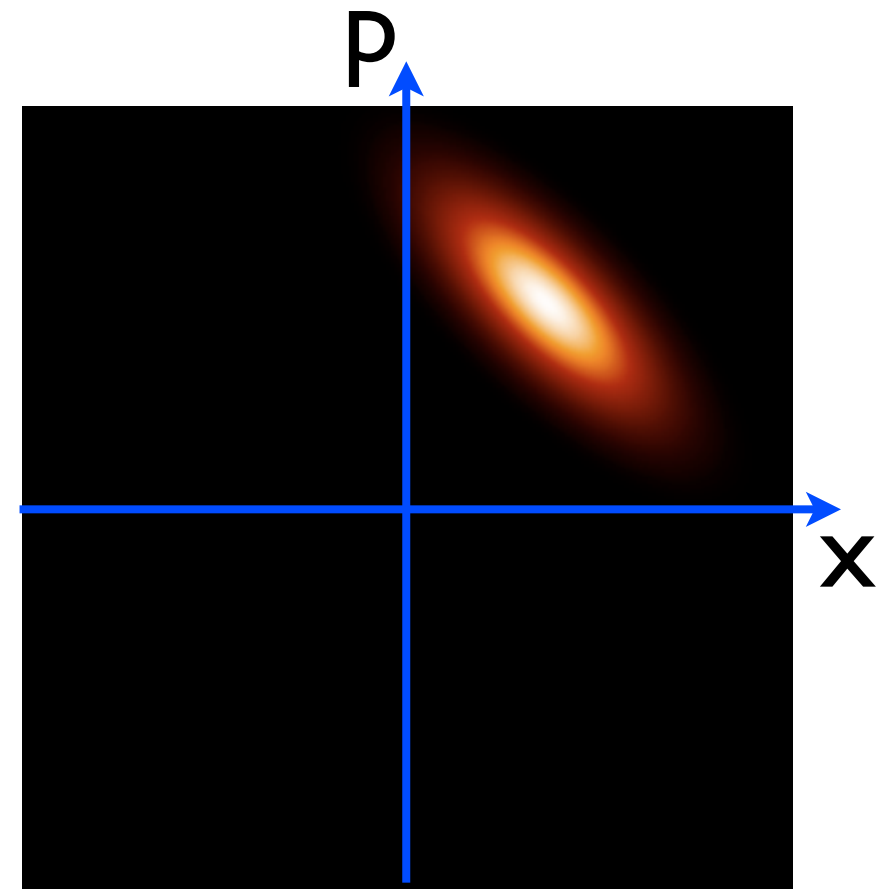


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**reconstruct  
mechanical  
Wigner density**

(quantum state tomography)

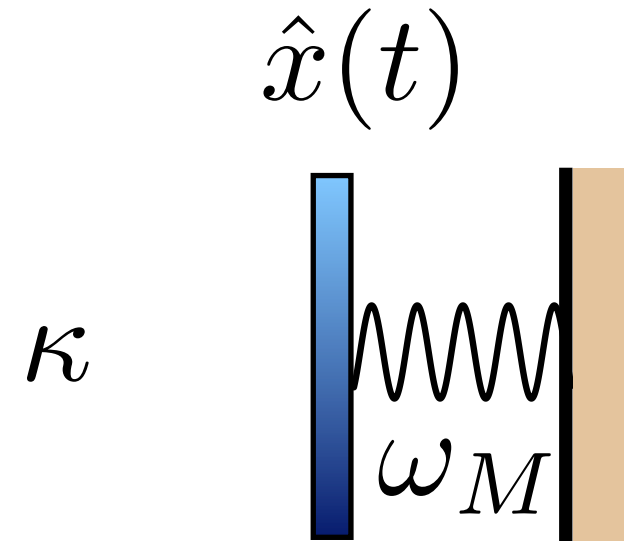
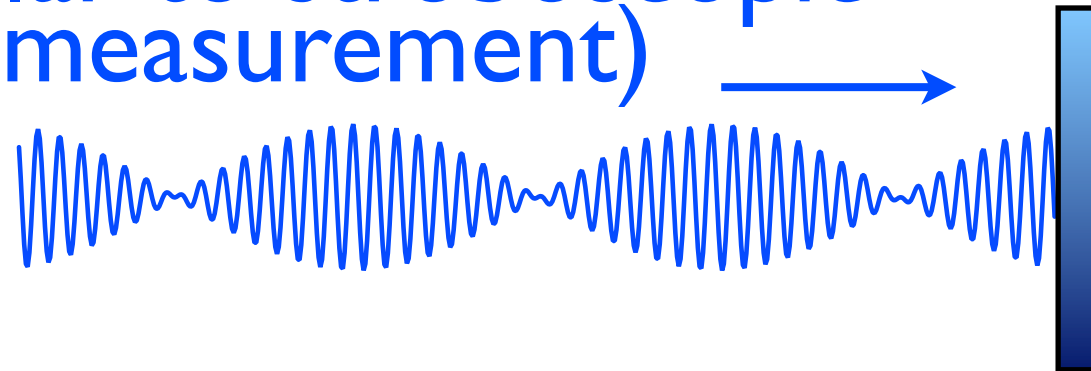
$$W(x, p) \propto \int dy e^{ipy/\hbar} \rho(x - y/2, x + y/2)$$



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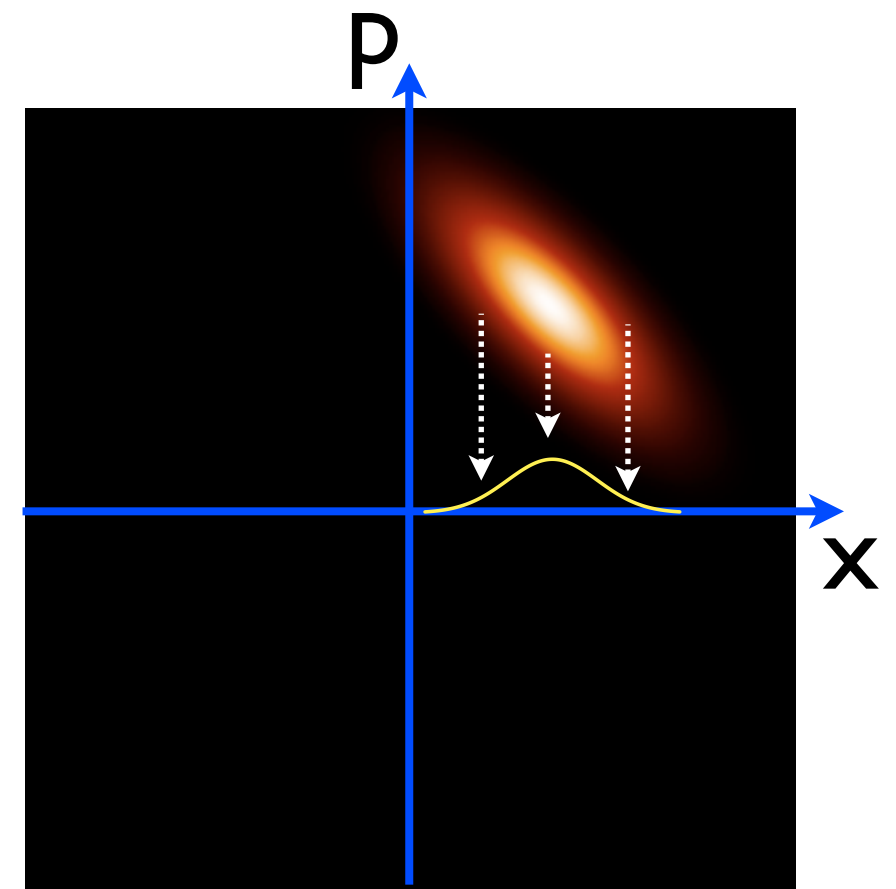


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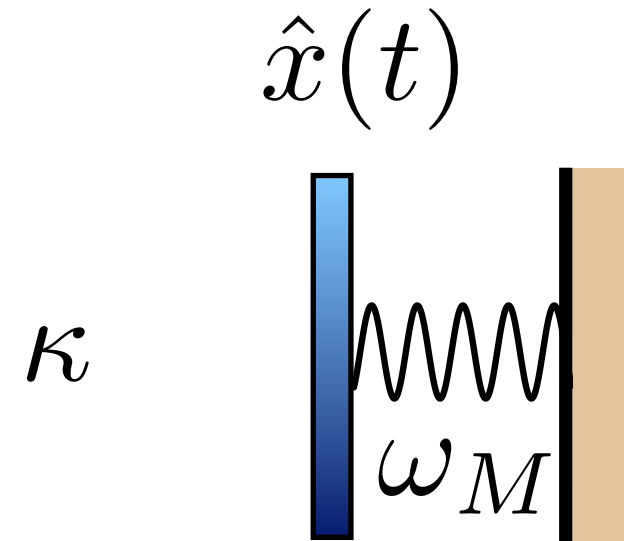
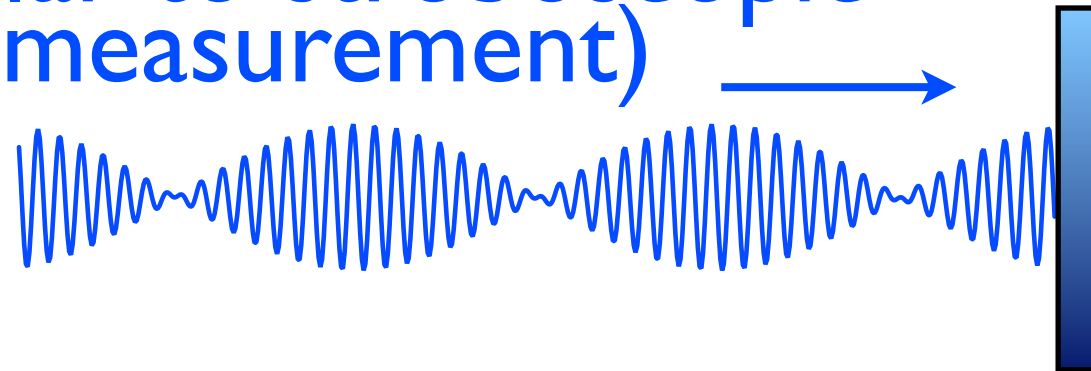
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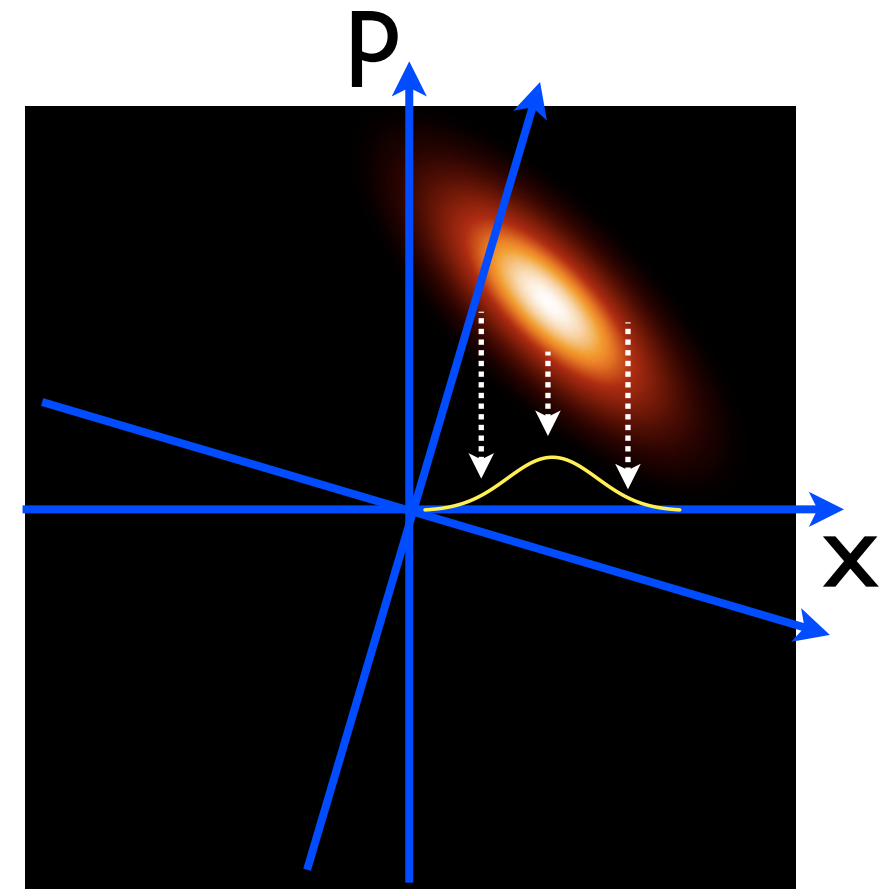


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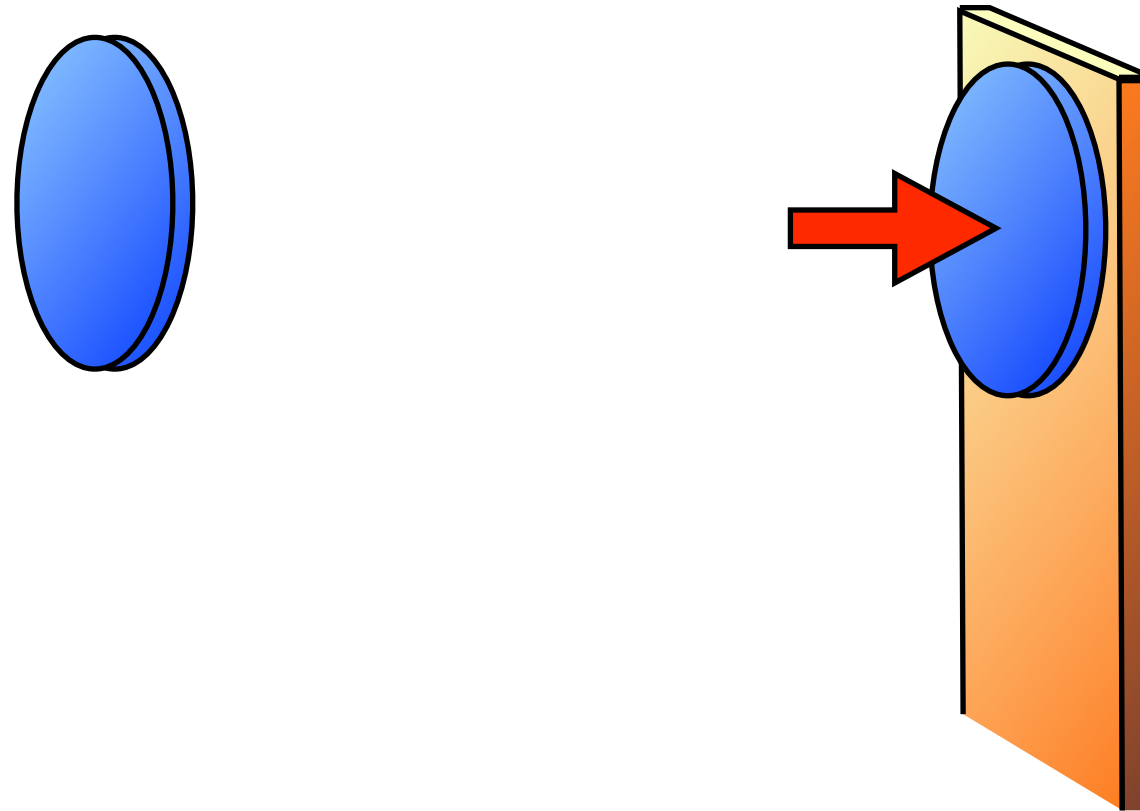
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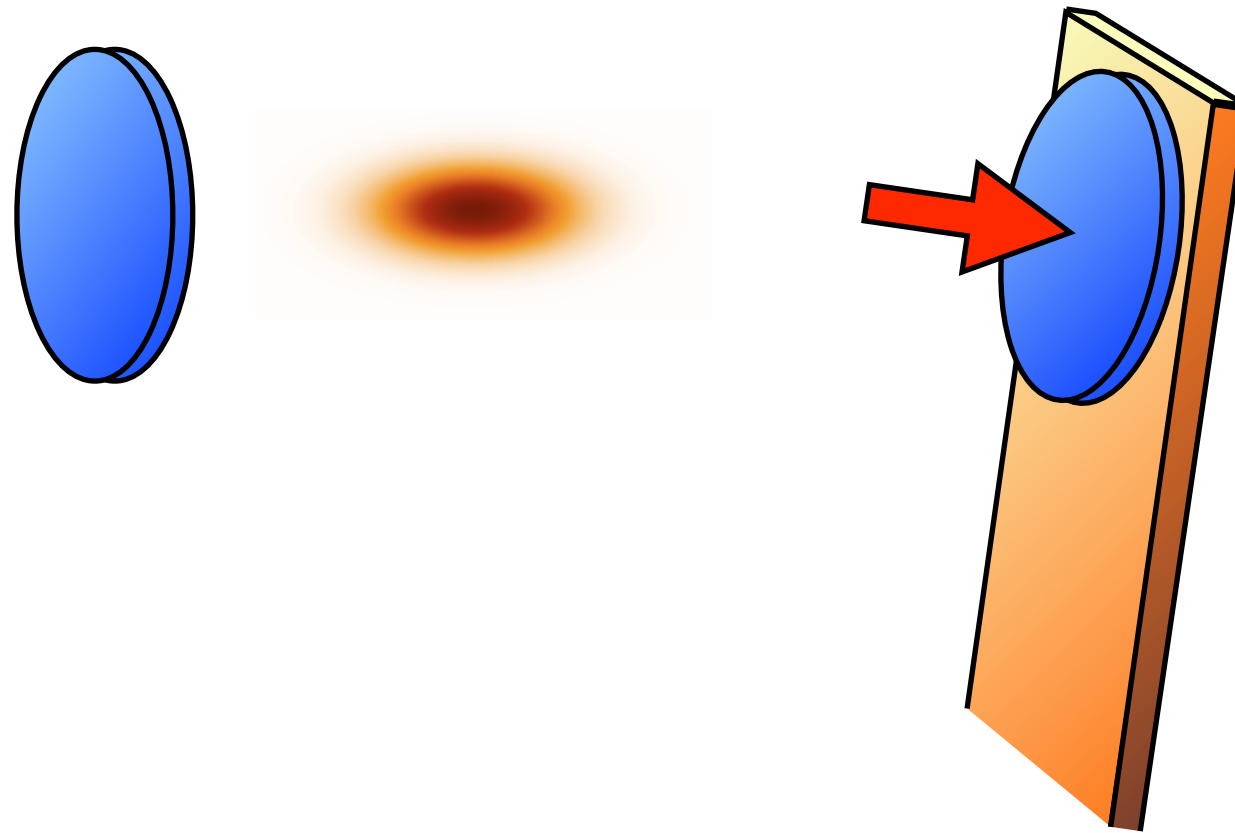


# Optomechanical entanglement



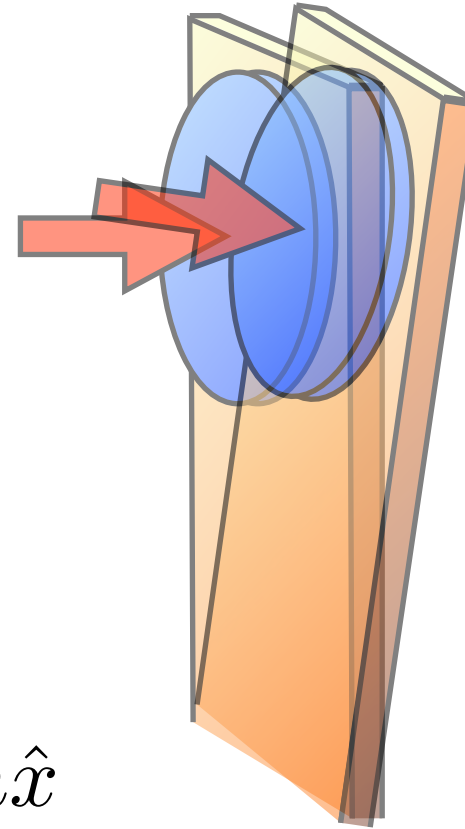
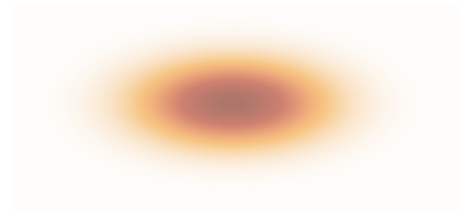
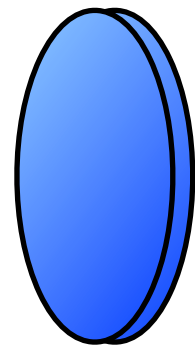
Bose, Jacobs, Knight 1997; Mancini et al. 1997

# Optomechanical entanglement



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# Optomechanical entanglement

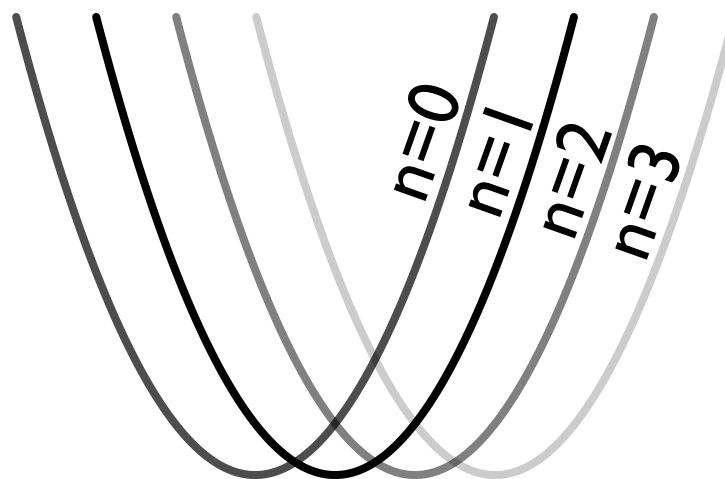


$$\hat{H} = \dots + g_0 \hat{n} \hat{x}$$

$$|\Psi\rangle = \sum_{n=0}^{\infty} c_n e^{-i\varphi_n(t)} |n\rangle \otimes |\alpha = \alpha_n(t)\rangle$$

coherent mechanical state

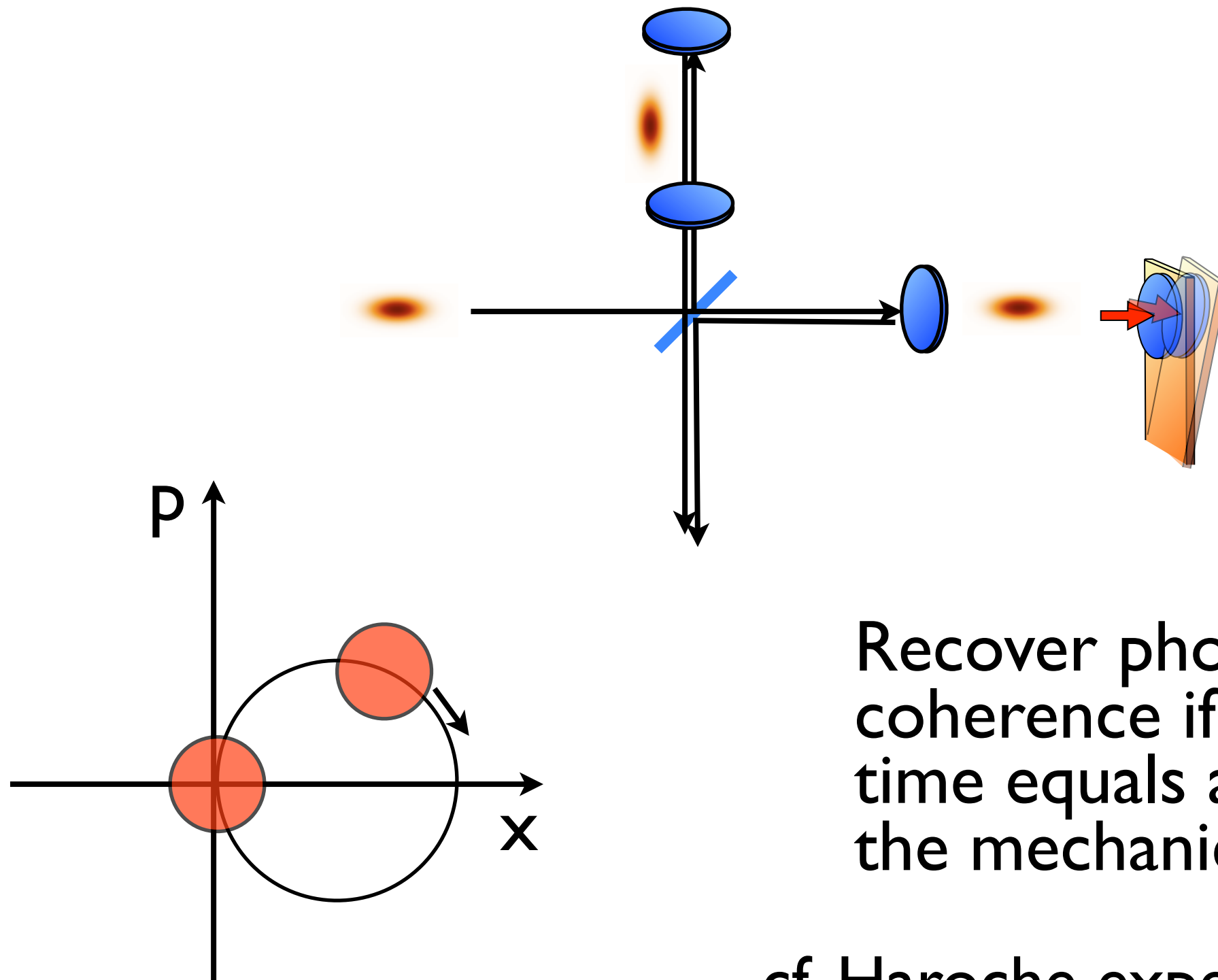
entangled state  
(light field/mechanics)



Bose, Jacobs, Knight 1997; Mancini et al. 1997



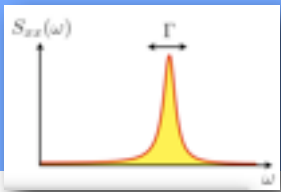
# Proposed optomechanical which-path experiment and quantum eraser



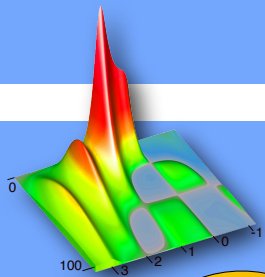
Recover photon coherence if interaction time equals a multiple of the mechanical period!

cf. Haroche experiments in 90s

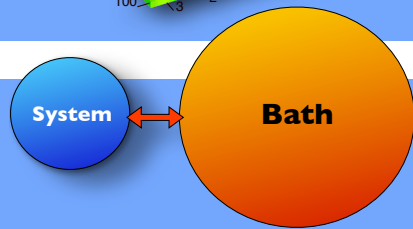
# Optomechanics (Outline)



Displacement detection

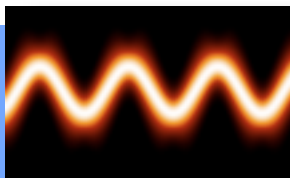


Linear optomechanics

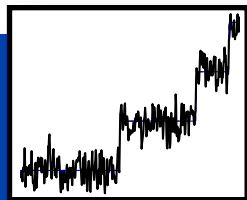


Nonlinear dynamics

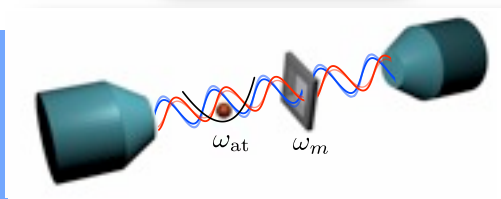
Quantum theory of cooling



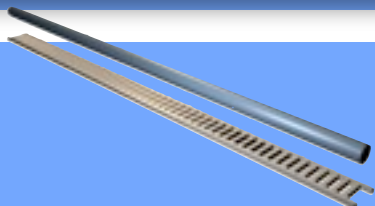
Interesting quantum states



Towards Fock state detection

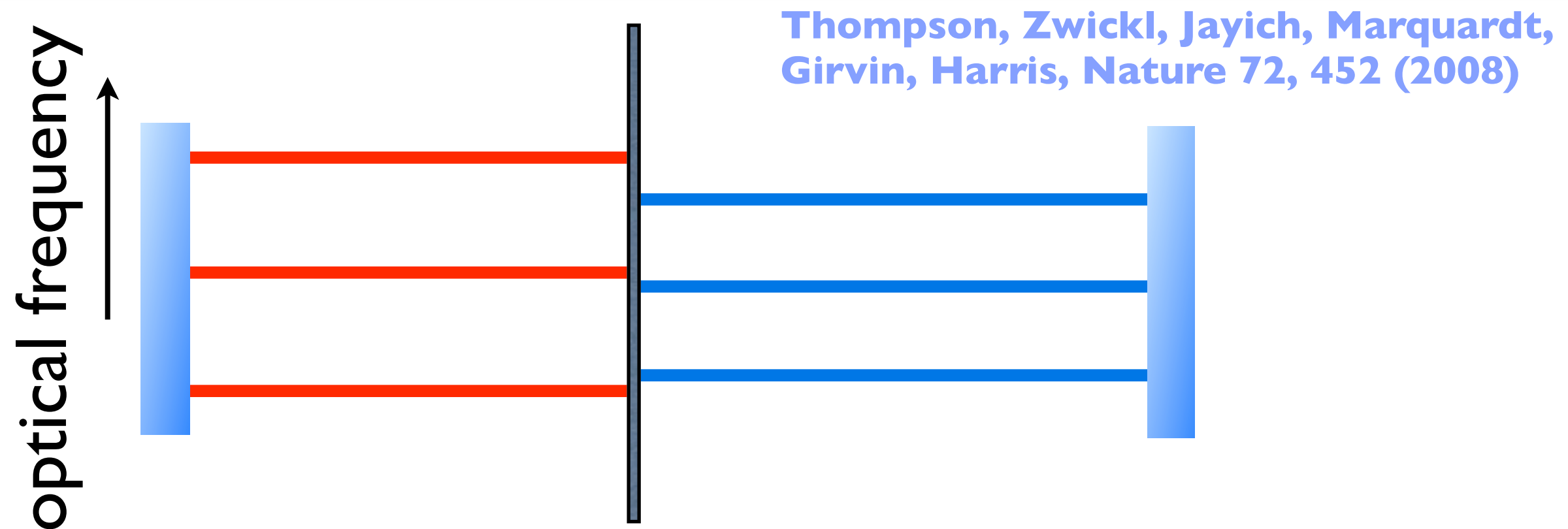


Hybrid systems: coupling to atoms

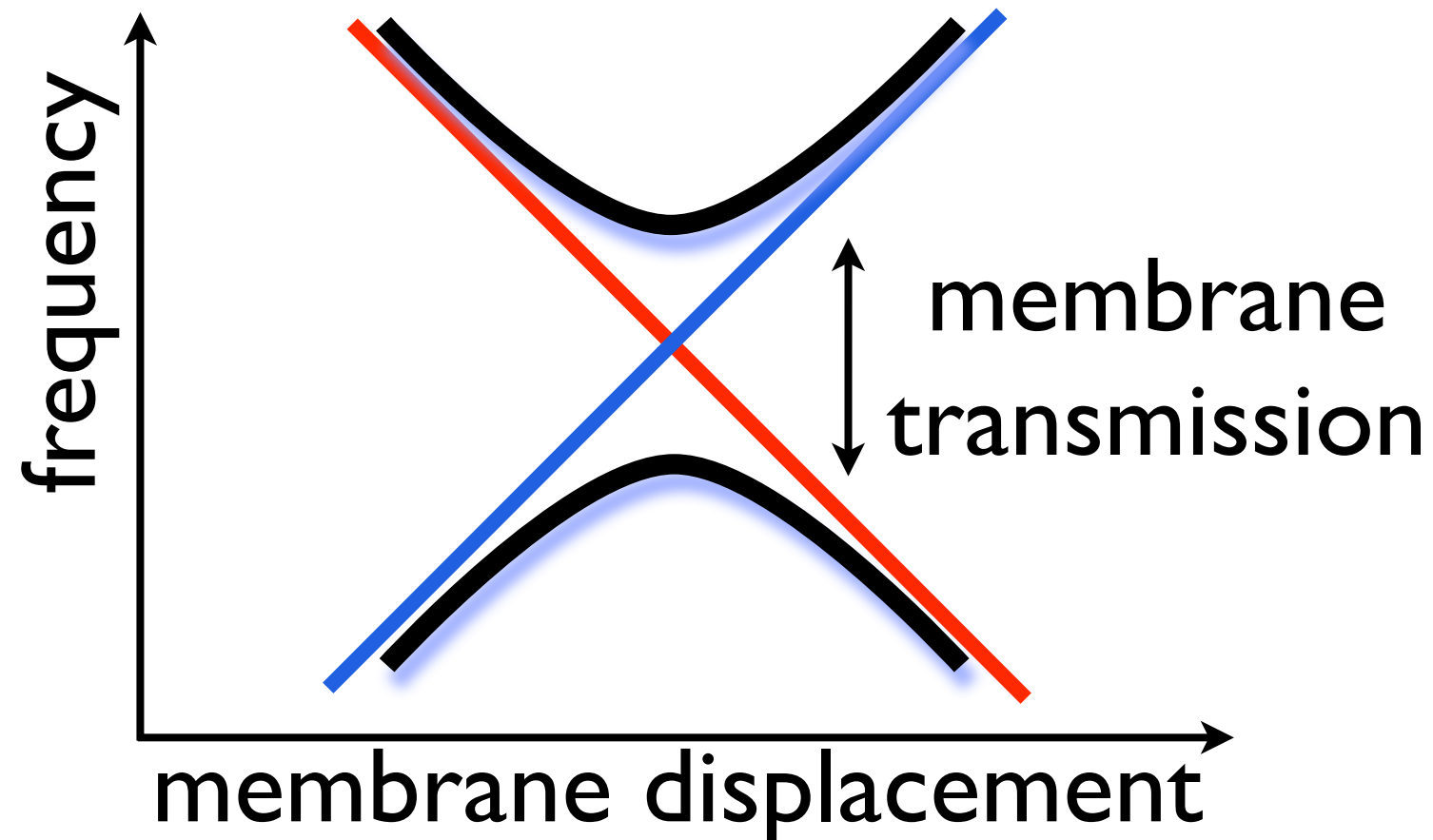
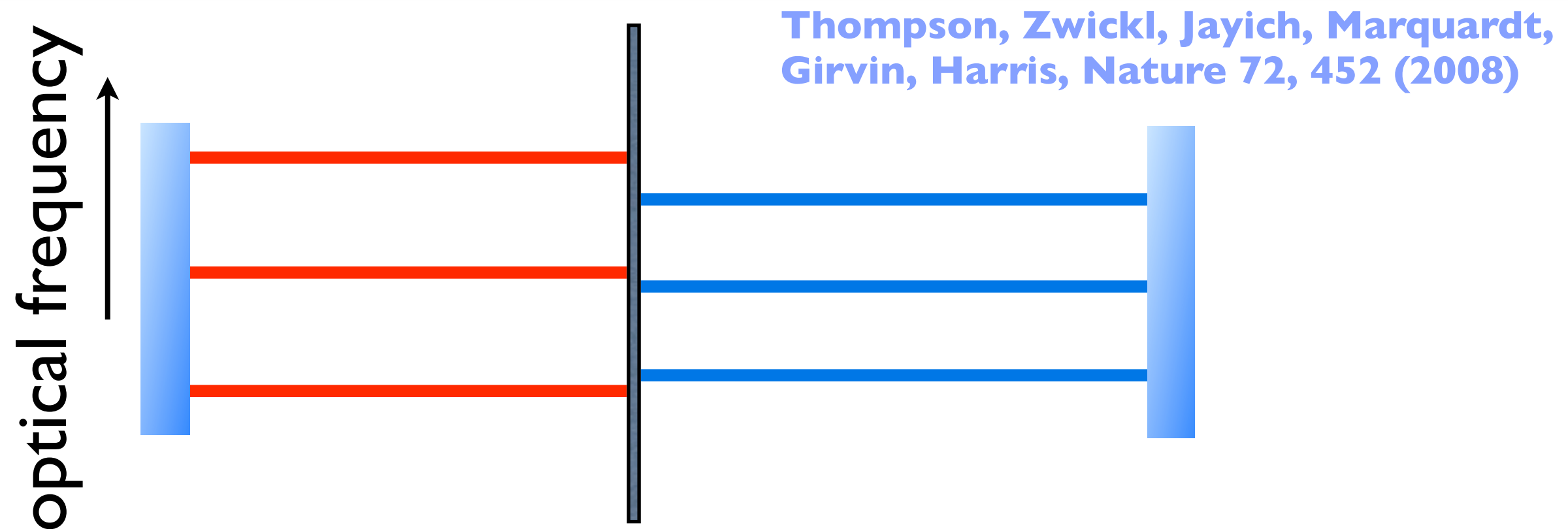


Optomechanical crystals & arrays

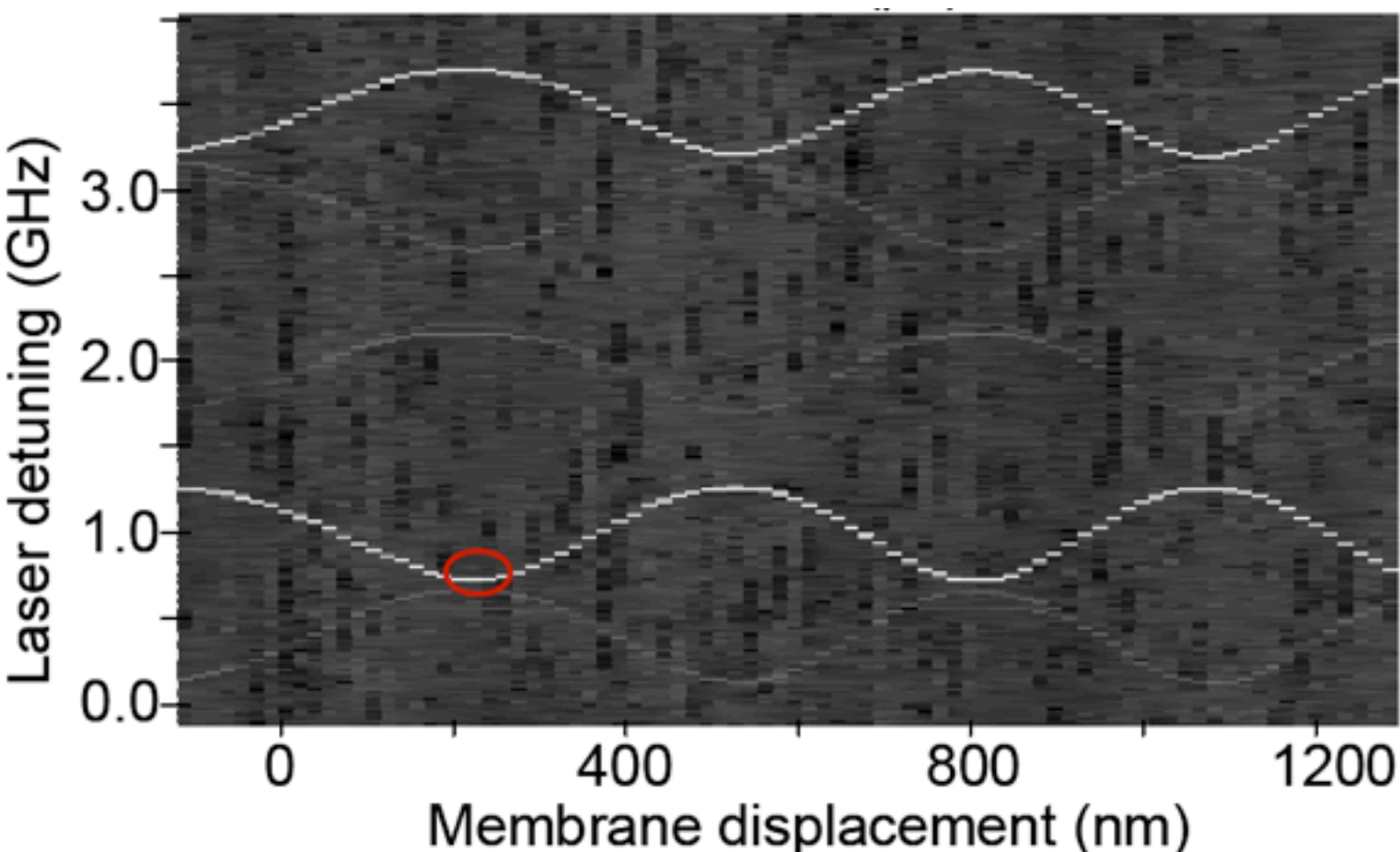
# “Membrane in the middle” setup



# “Membrane in the middle” setup



# Experiment (Harris group, Yale)



Mechanical frequency:

$$\omega_M = 2\pi \cdot 134 \text{ kHz}$$

Mechanical quality factor:

$$Q = 10^6 \div 10^7$$

Current optical finesse:

$$7000 \div 15000 \text{ (} 5 \cdot 10^5 \text{)}$$

[almost sideband regime]

Optomechanical cooling  
from **300K** to **7mK**

Thompson, Zwickl, Jayich, Marquardt,  
Girvin, Harris, *Nature* 72, 452 (2008)

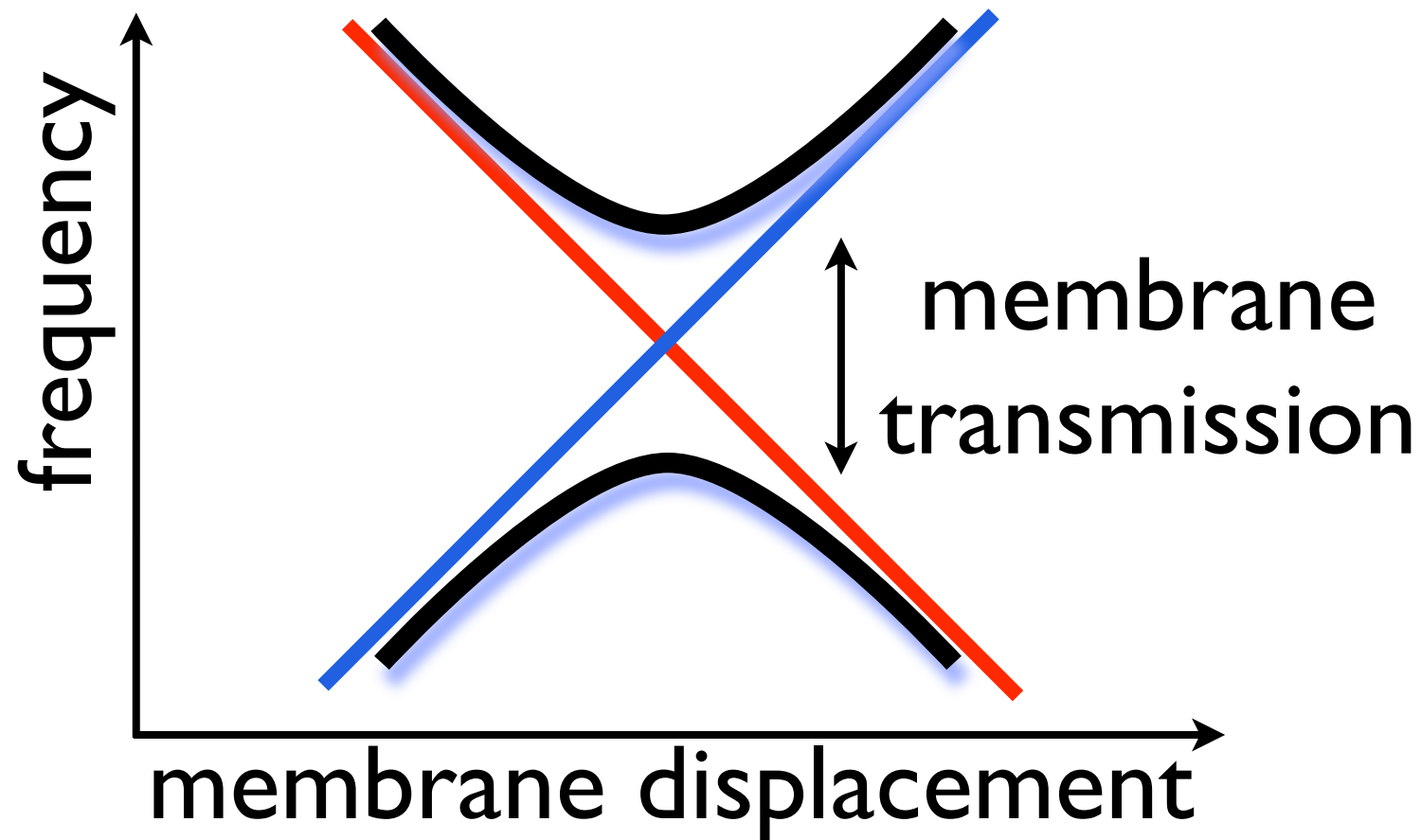
# **Towards Fock state detection of a macroscopic object**

Detection of displacement  $x$ : *not* what we need!



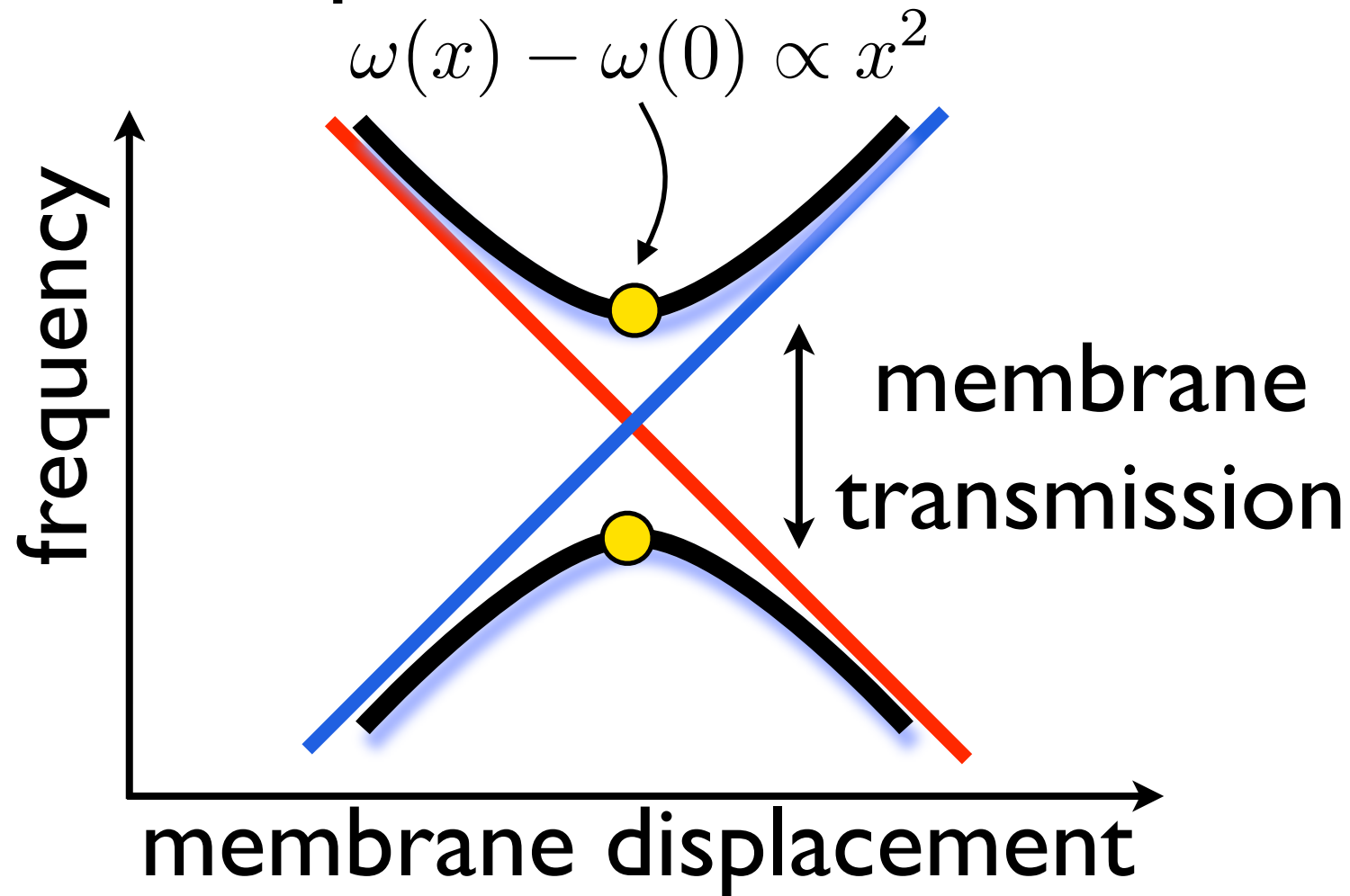
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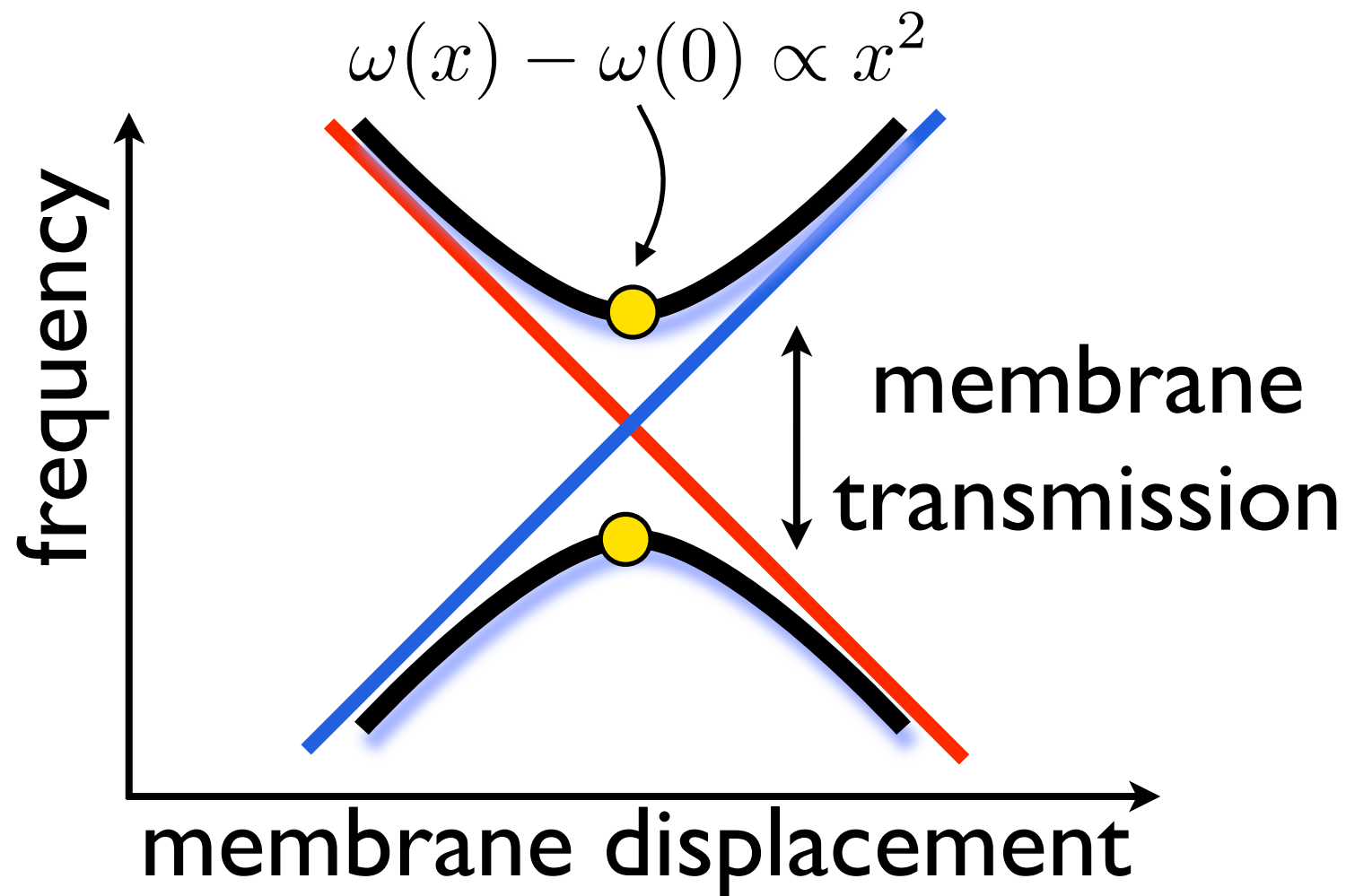


# Towards Fock state detection of a macroscopic object

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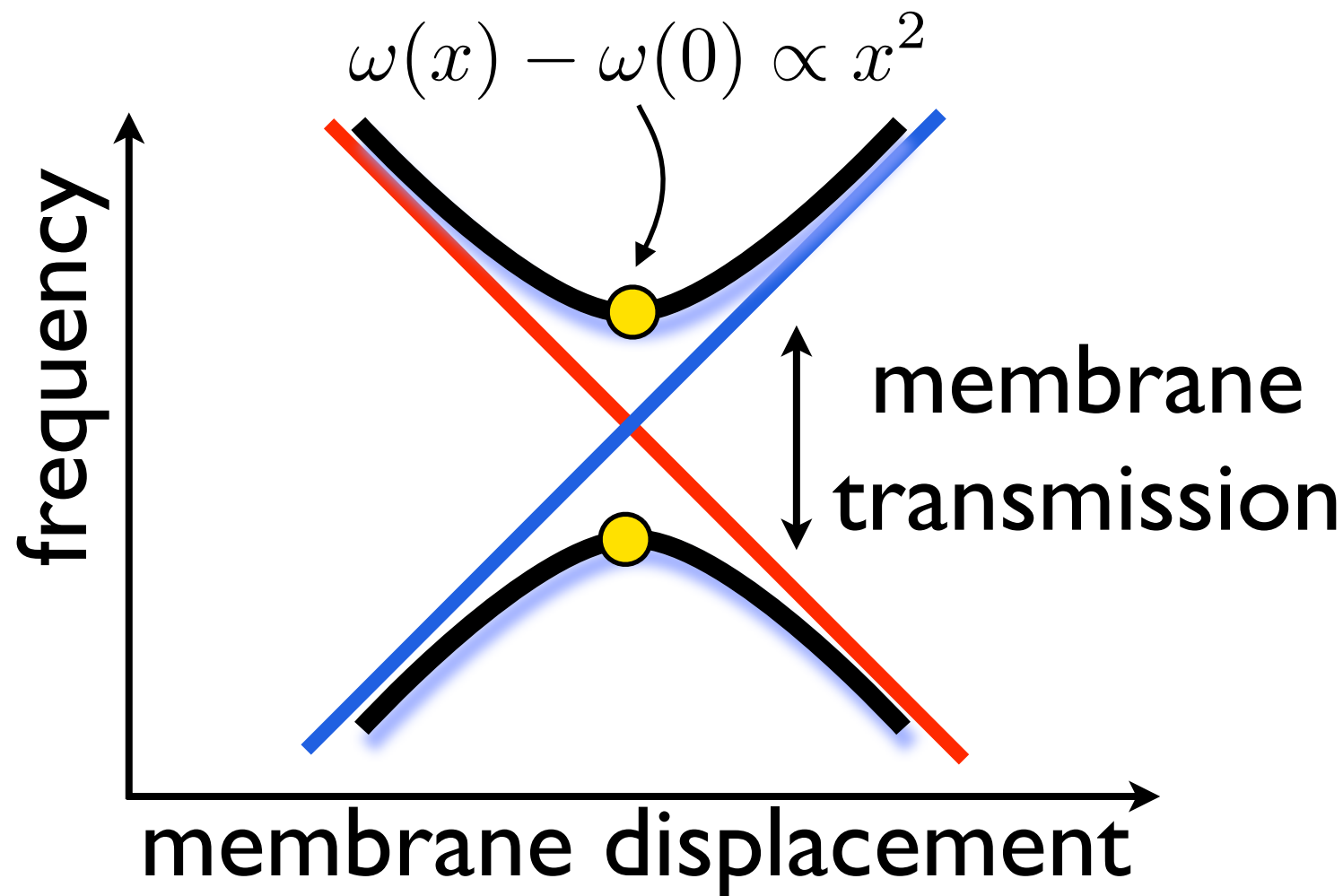
# Towards Fock state detection of a macroscopic object



phase shift of measurement beam:

$$\hat{\theta} \propto \hat{x}(t)^2 \propto (\hat{b}(t) + \hat{b}^\dagger(t))^2 = \hat{b}^2 e^{-i2\omega_M t} + \hat{b}^{\dagger 2} e^{+i2\omega_M t} + \hat{b}^\dagger \hat{b} + \hat{b} \hat{b}^\dagger$$

# Towards Fock state detection of a macroscopic object



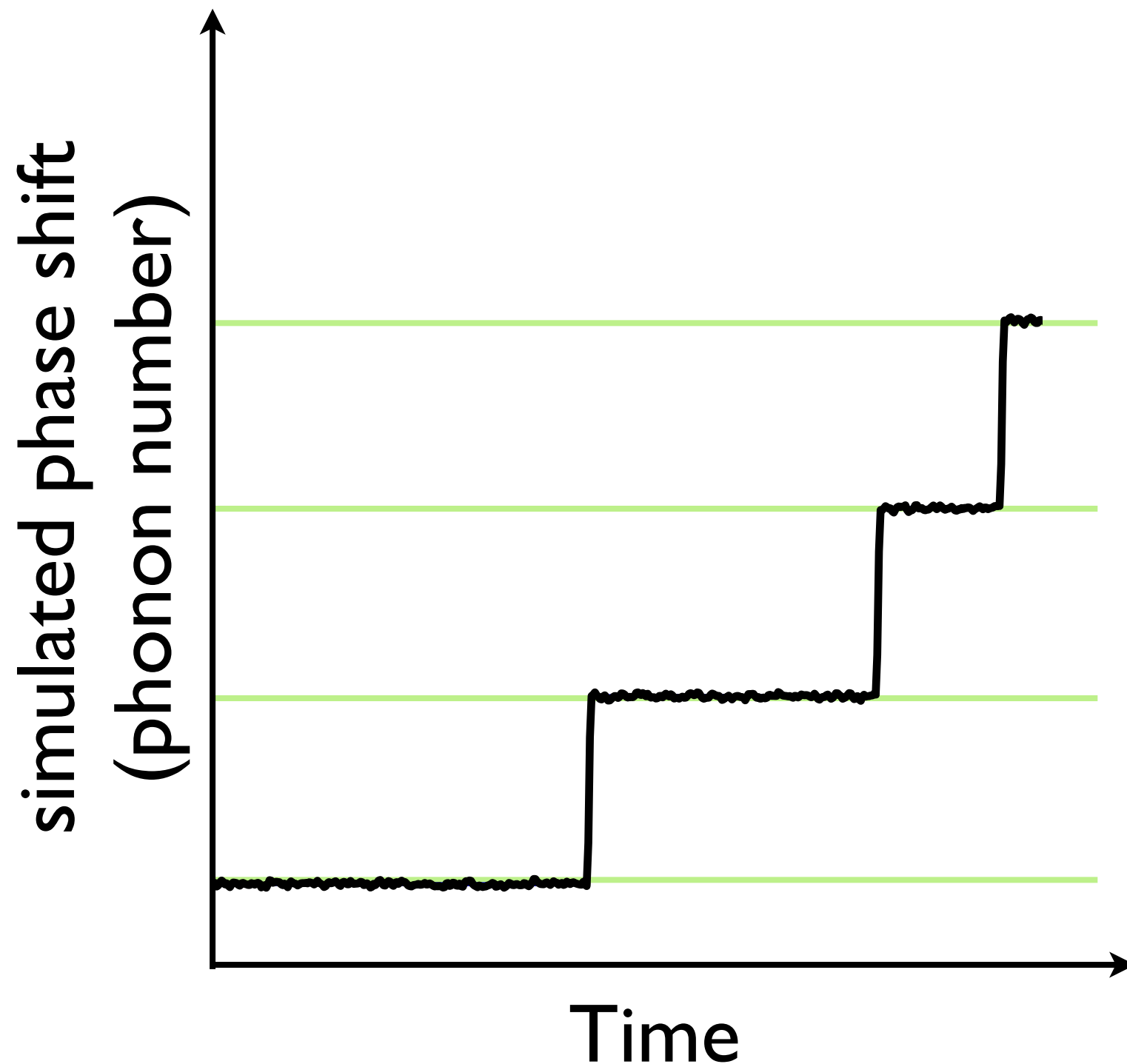
phase shift of measurement beam:

$$\overline{\hat{\theta}} \propto \overline{\hat{x}(t)^2} \propto \overline{(\hat{b}(t) + \hat{b}^\dagger(t))^2} \approx 2\underbrace{\hat{b}^\dagger \hat{b}} + 1$$

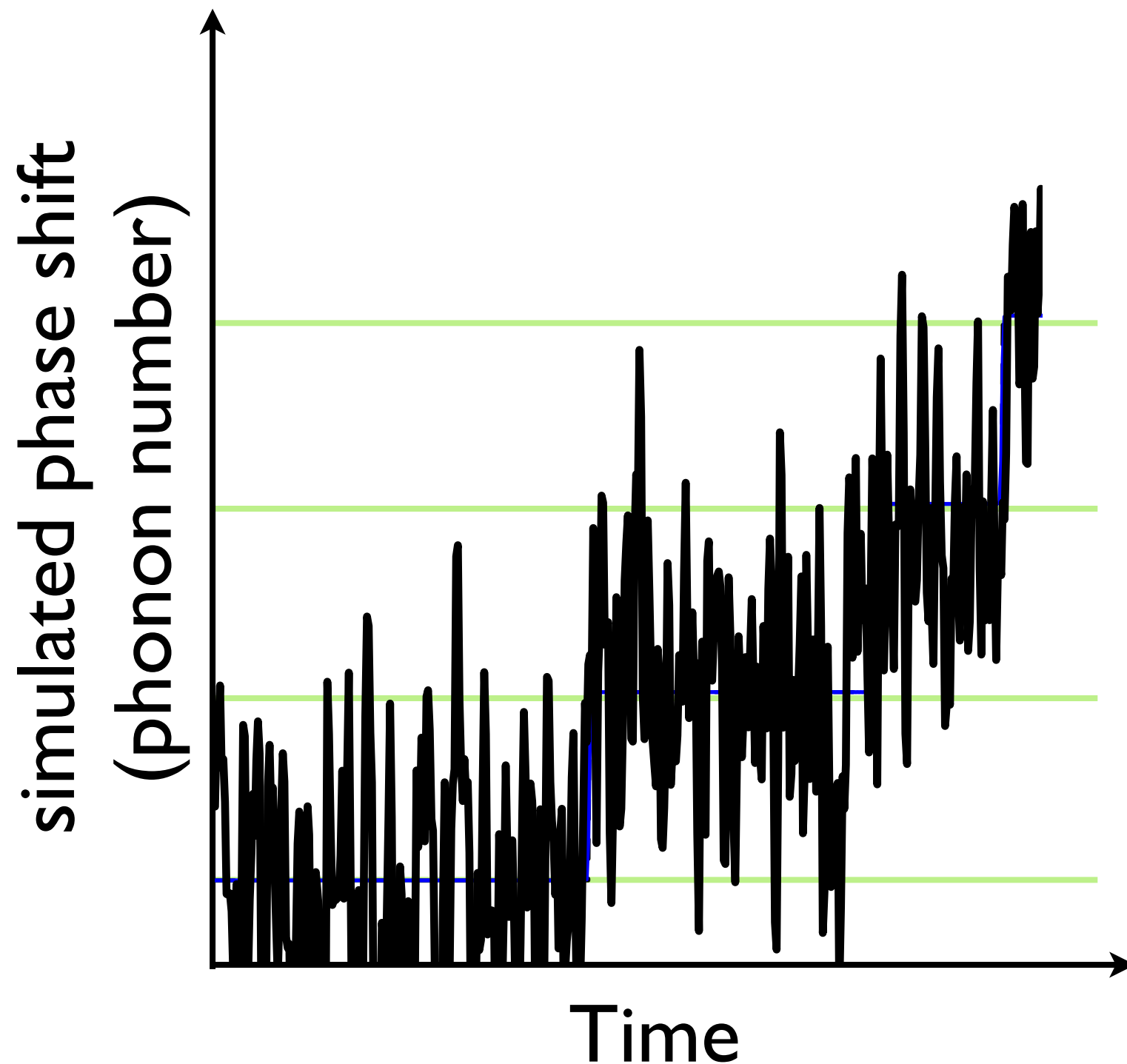
(Time-average over  
cavity ring-down time)

**QND measurement  
of phonon number!**

# Towards Fock state detection of a macroscopic object



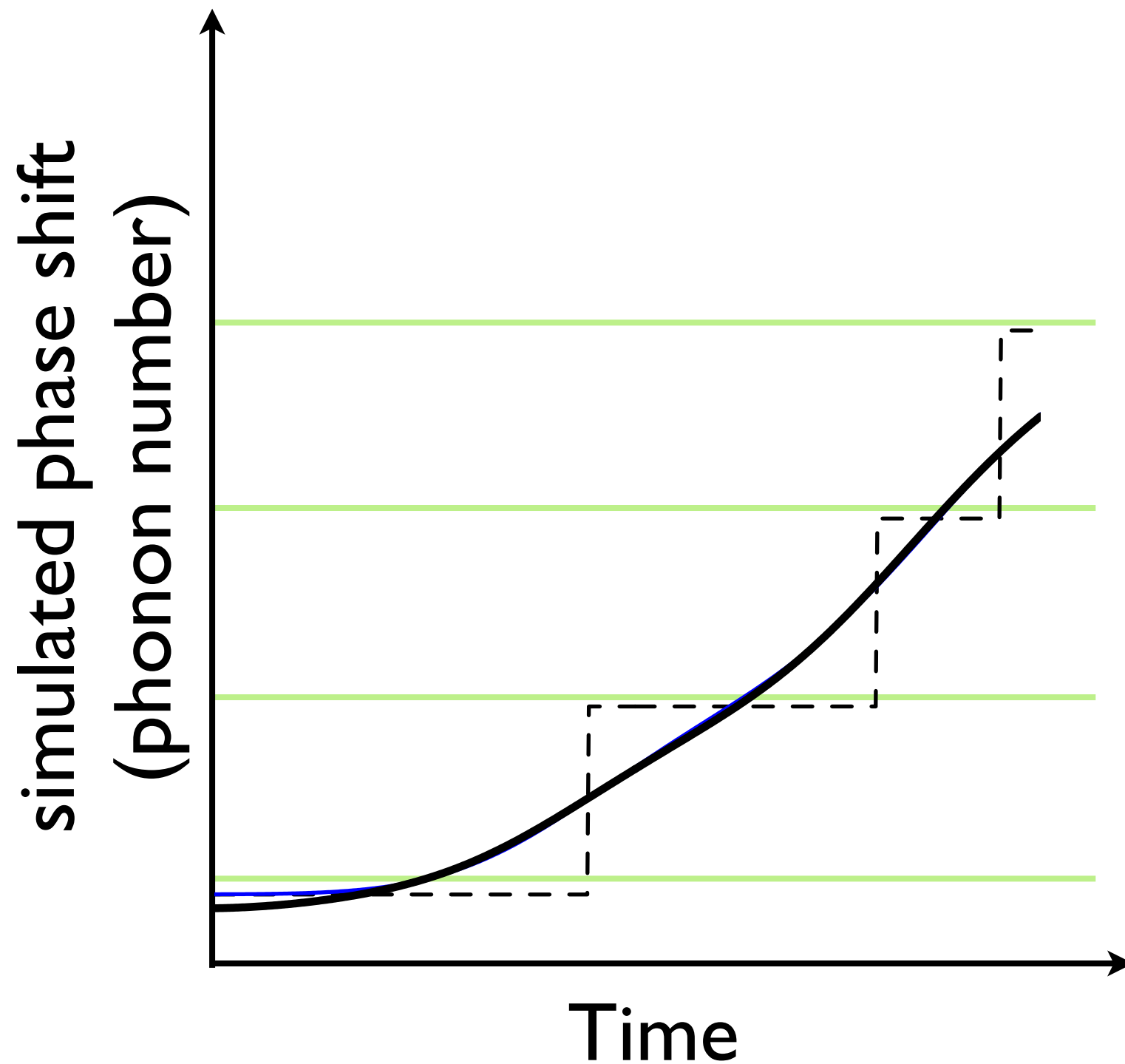
# Towards Fock state detection of a macroscopic object



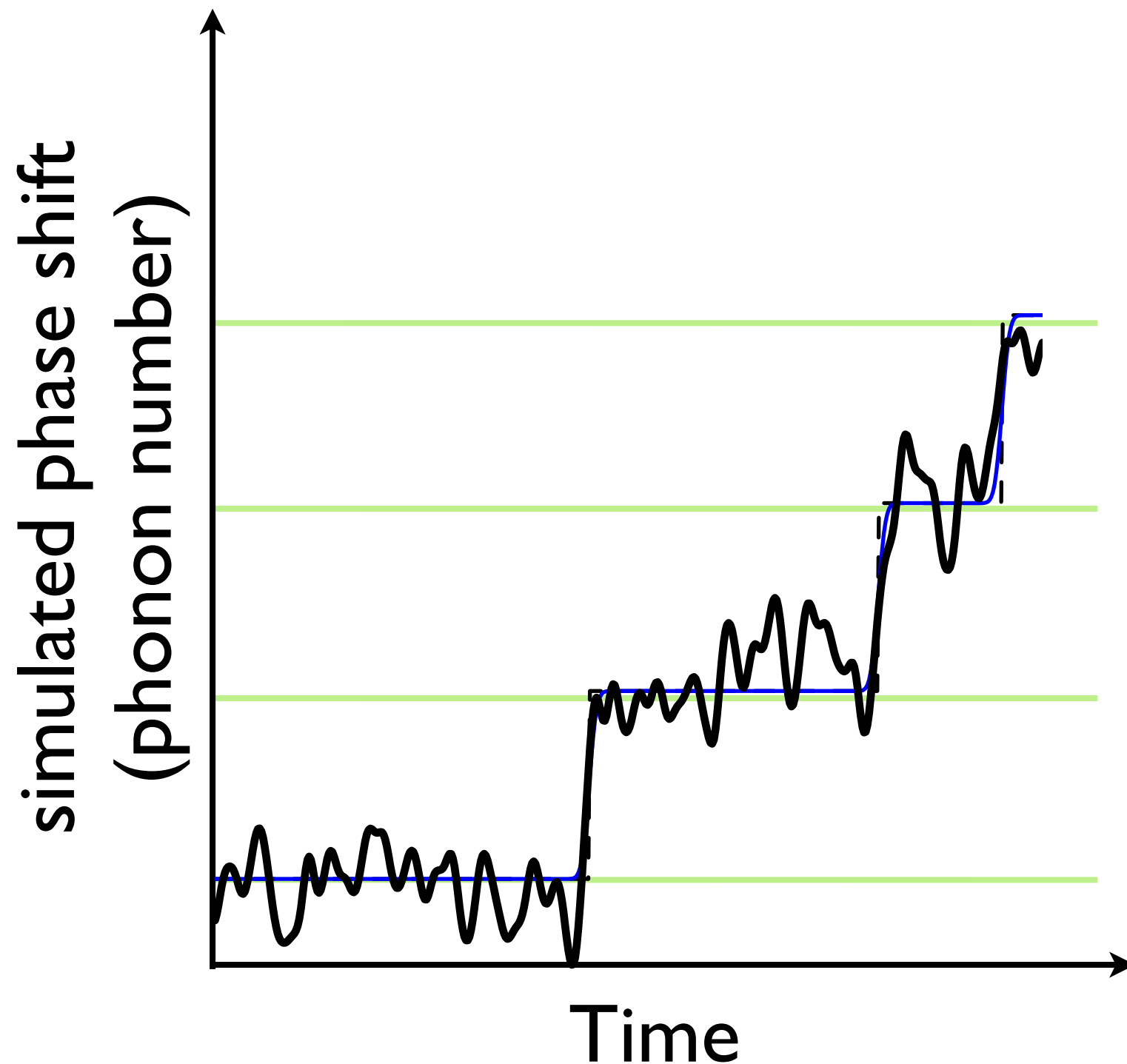
Thompson, Zwickl, Jayich, FM,  
Girvin, Harris, Nature 72, 452 (2008)



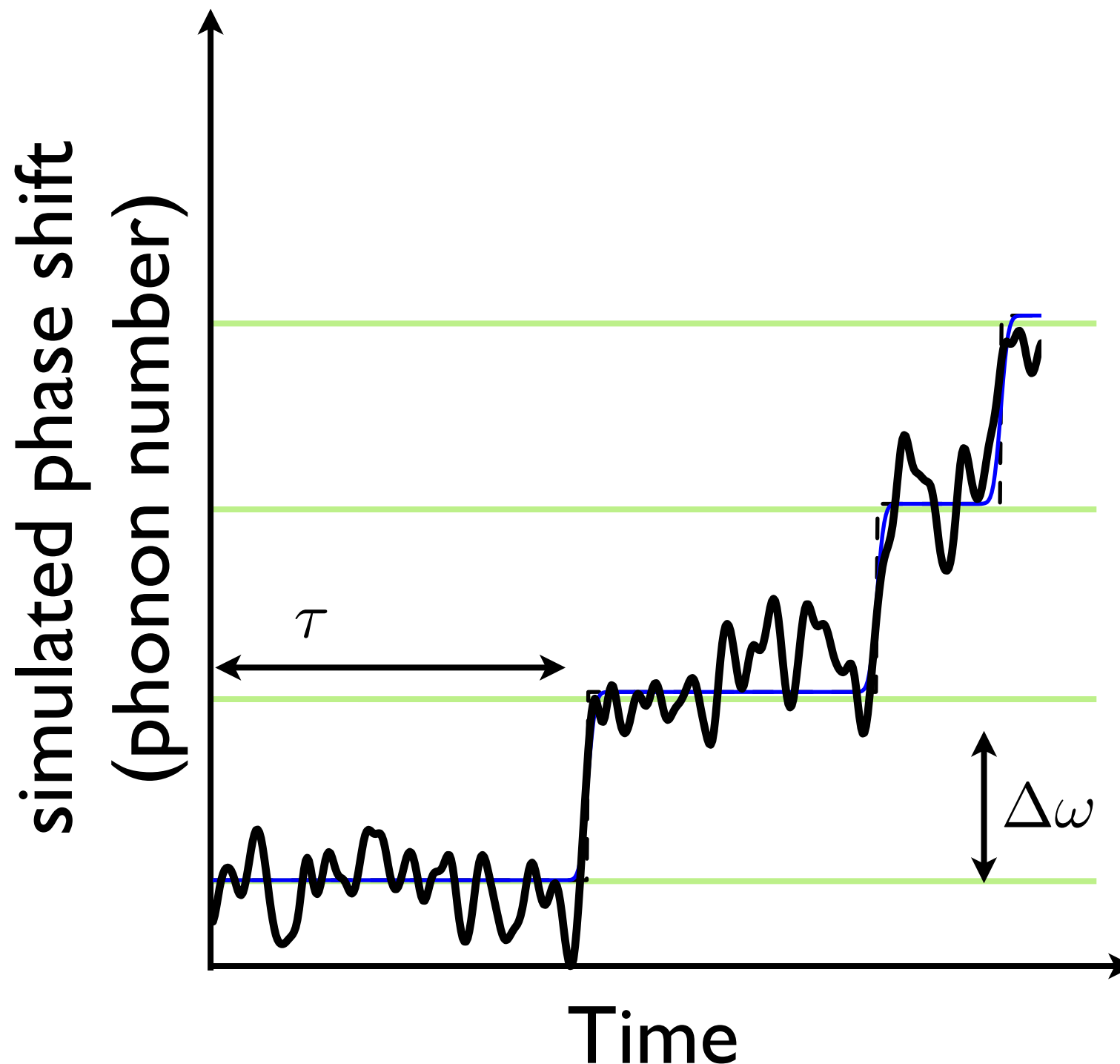
# Towards Fock state detection of a macroscopic object



# Towards Fock state detection of a macroscopic object



# Towards Fock state detection of a macroscopic object



**Signal-to-noise**

**ratio:** 
$$\frac{\tau \Delta\omega^2}{S_\omega}$$

Optical freq. shift  
per phonon:

$$\Delta\omega = x_{\text{ZPF}}^2 \omega''$$

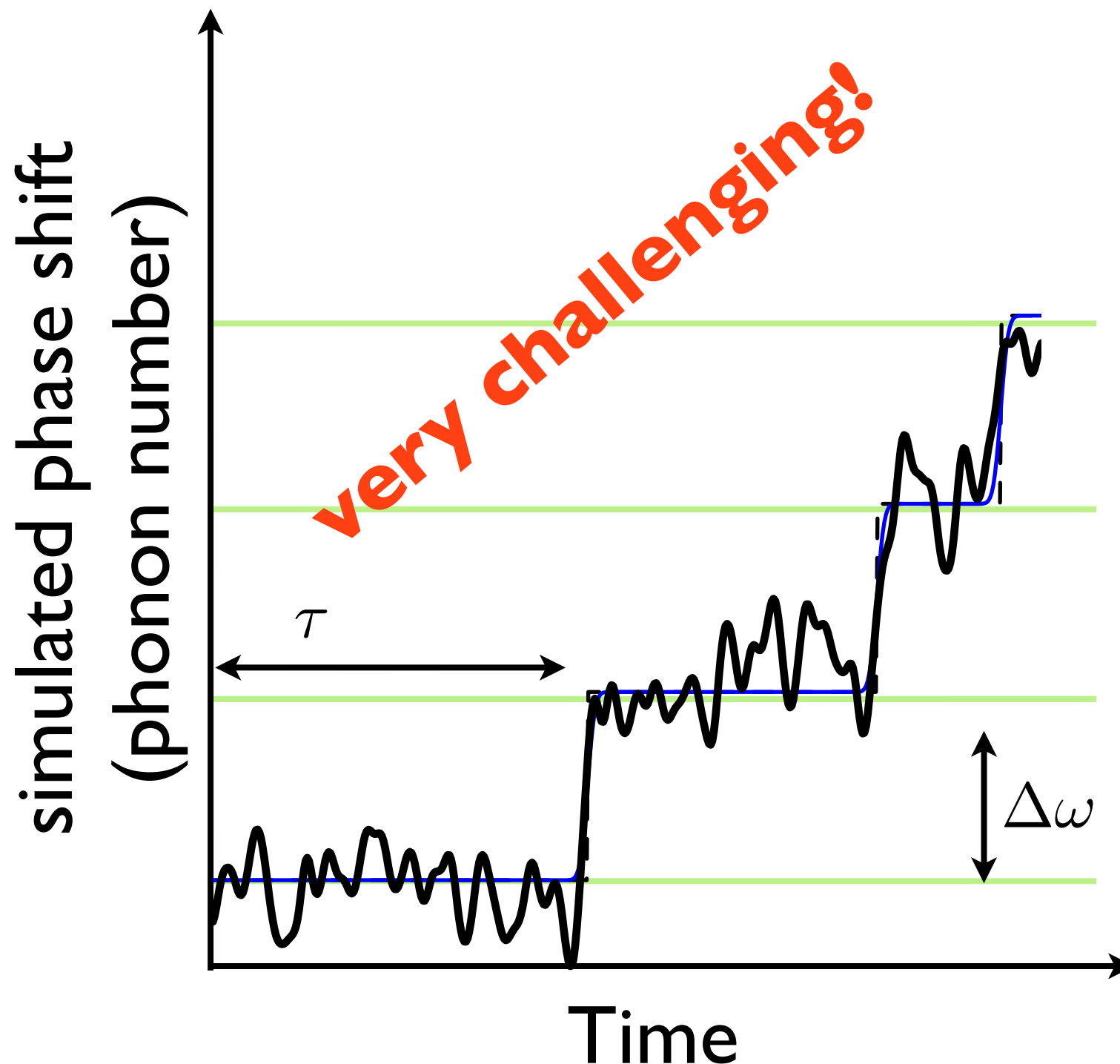
Noise power of  
freq. measurement:

$$S_\omega = \frac{\kappa}{16\bar{n}_{\text{cavity}}}$$

Ground state lifetime:

$$\frac{1}{\tau} = \Gamma \bar{n}_{\text{thermal}}$$

# Towards Fock state detection of a macroscopic object



**Signal-to-noise**

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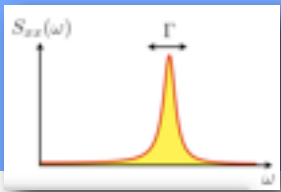
Noise power of  
freq. measurement:

$$S_\omega = \frac{\kappa}{16\bar{n}_{\text{cavity}}}$$

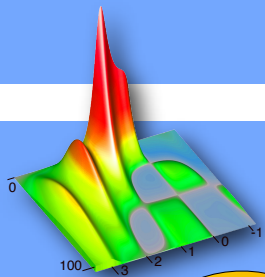
Ground state lifetime:

$$\frac{1}{\tau} = \Gamma \bar{n}_{\text{thermal}}$$

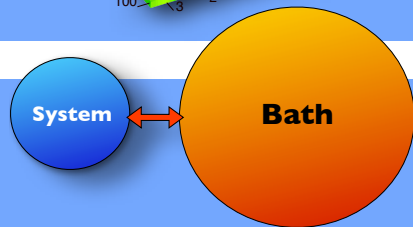
# Optomechanics (Outline)



Displacement detection

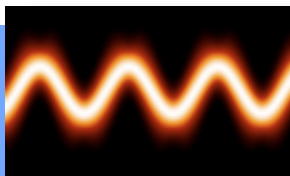


Linear optomechanics

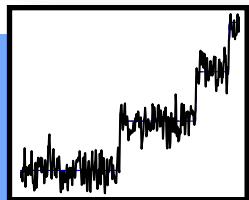


Nonlinear dynamics

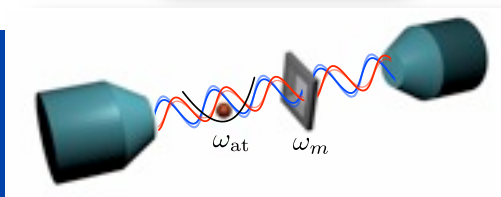
Quantum theory of cooling



Interesting quantum states



Towards Fock state detection



Hybrid systems: coupling to atoms



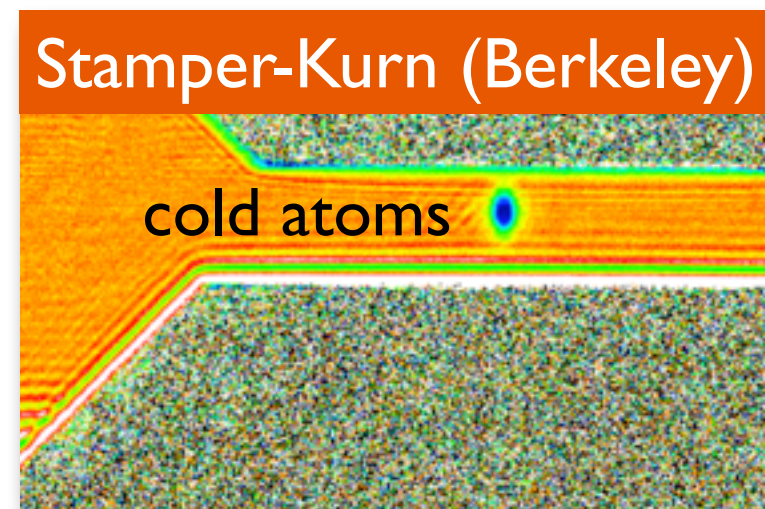
Optomechanical crystals & arrays

# Atom-membrane coupling

Note: Existing works simulate optomechanical effects using cold atoms

K.W. Murch, K. L. Moore, S. Gupta, and D. M. Stamper-Kurn, Nature Phys. **4**, 561 (2008).

F. Brennecke, S. Ritter, T. Donner, and T. Esslinger, Science **322**, 235 (2008).



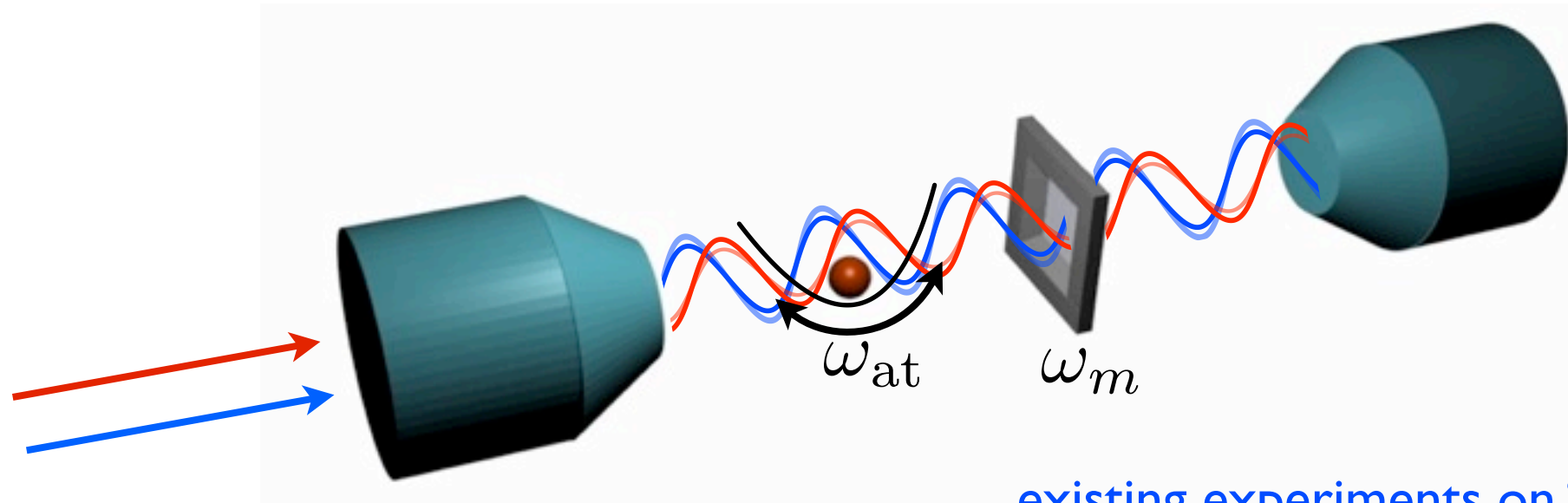
...profit from small mass of atomic cloud

Here: Coupling a single atom to a macroscopic mechanical object

Challenge: huge mass ratio



# Strong atom-membrane coupling via the light field



existing experiments on “optomechanics with cold atoms”: labs of Dan-Stamper Kurn (Berkeley) and Tilman Esslinger (ETH)

collaboration:

LMU (M. Ludwig, FM, P. Treutlein),

Innsbruck (K. Hammerer, C. Genes, M. Wallquist, P. Zoller),

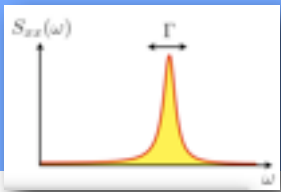
Boulder (J. Ye), Caltech (H. J. Kimble)

[Hammerer et al., PRL 2009](#)

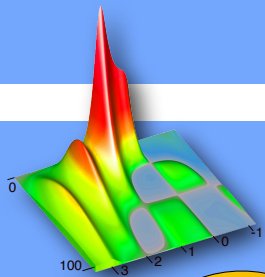
Goal:

$$\hat{H} = \underbrace{\hbar\omega_{at}\hat{a}^\dagger\hat{a}}_{\text{atom}} + \underbrace{\hbar\omega_m\hat{b}^\dagger\hat{b}}_{\text{membrane}} + \underbrace{\hbar G_{\text{eff}}(\hat{a}^\dagger + \hat{a})(\hat{b}^\dagger + \hat{b})}_{\text{atom-membrane coupling}}$$

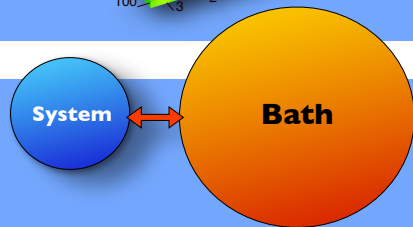
# Optomechanics (Outline)



Displacement detection

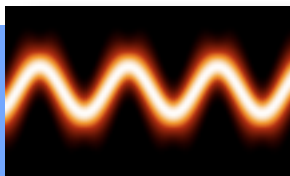


Linear optomechanics

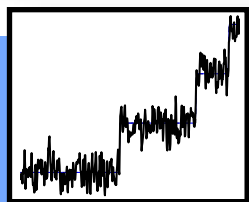


Nonlinear dynamics

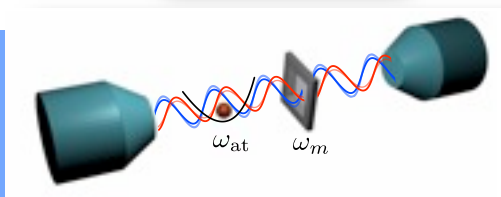
Quantum theory of cooling



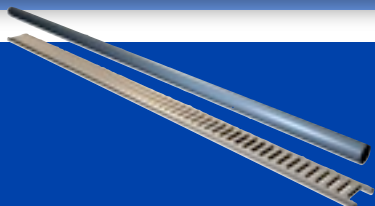
Interesting quantum states



Towards Fock state detection



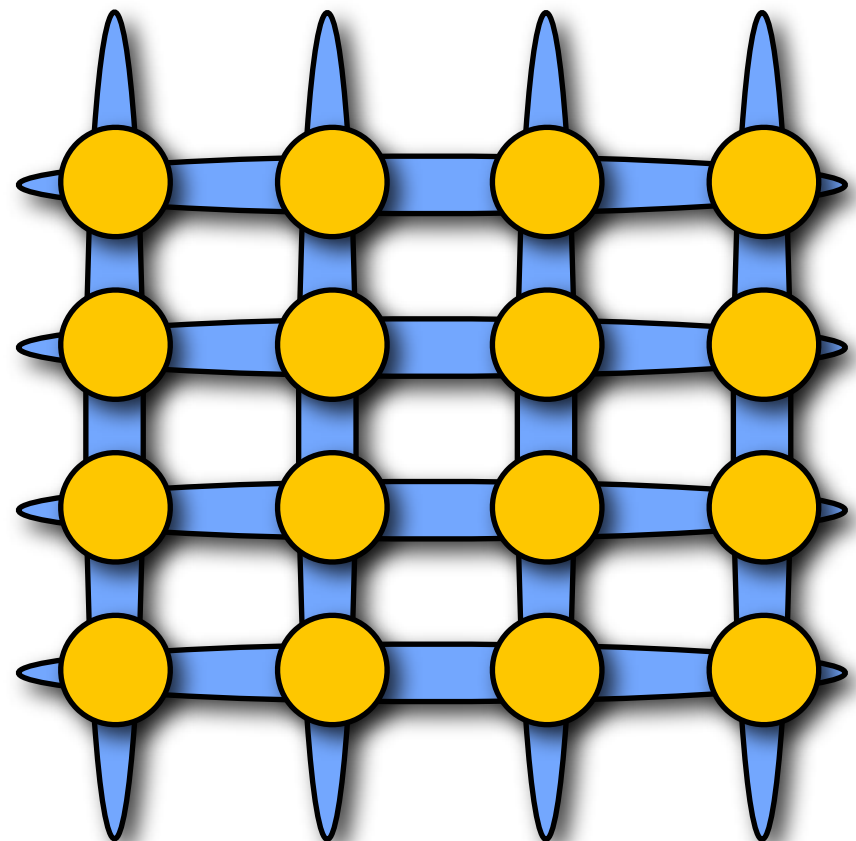
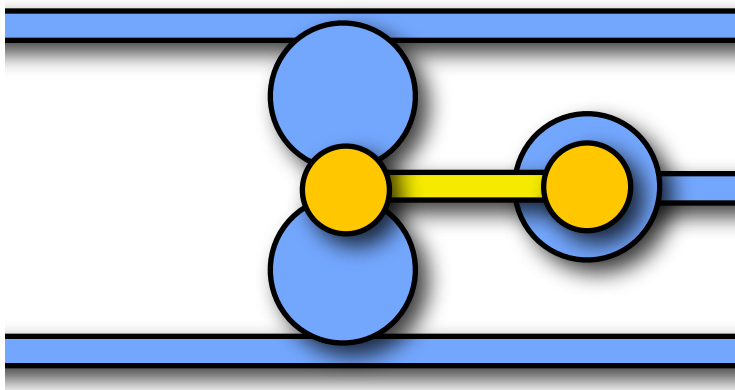
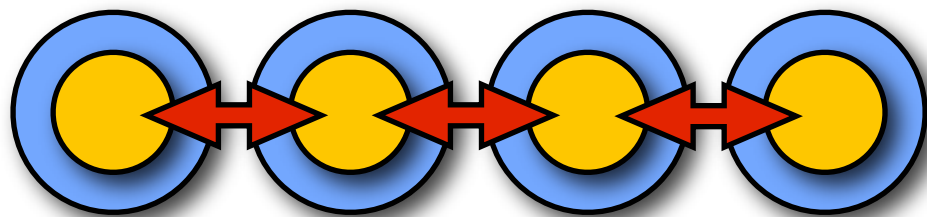
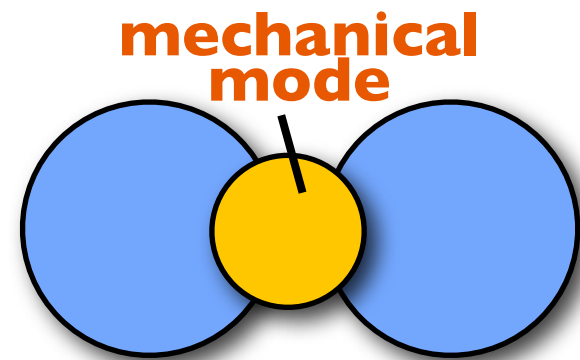
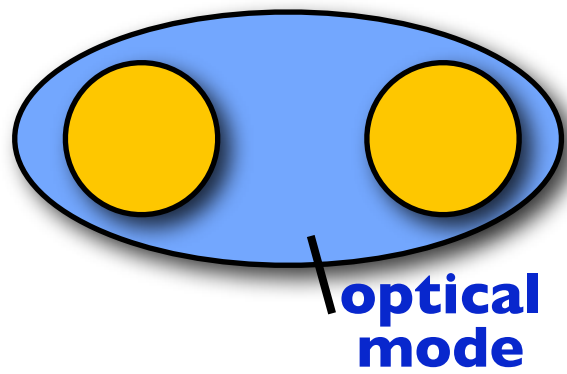
Hybrid systems: coupling to atoms



Optomechanical crystals & arrays

# Many modes

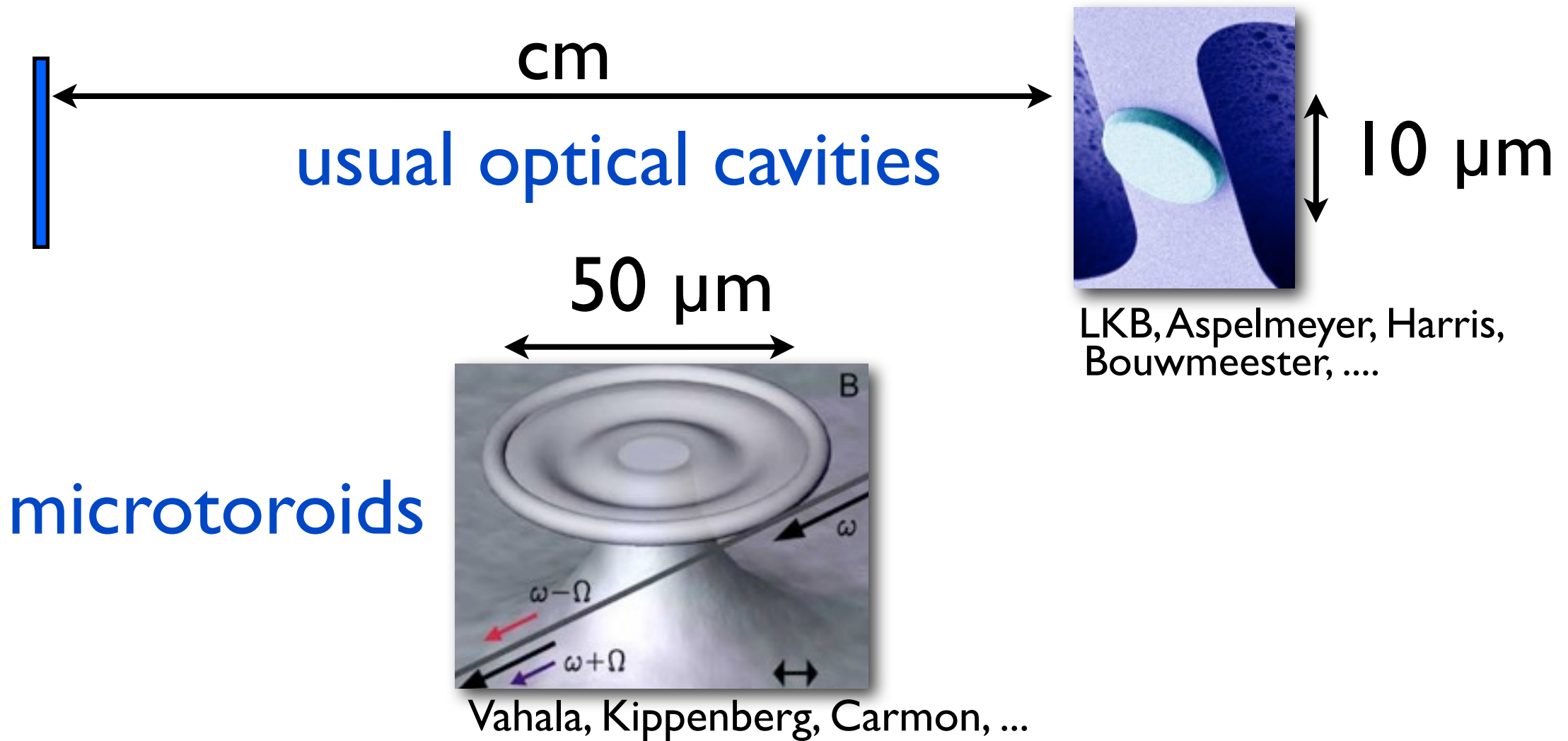
8



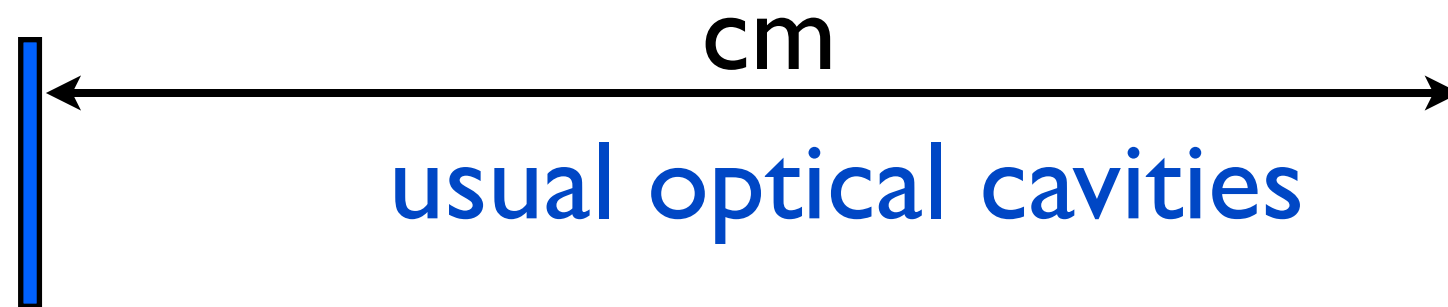
# Scaling down



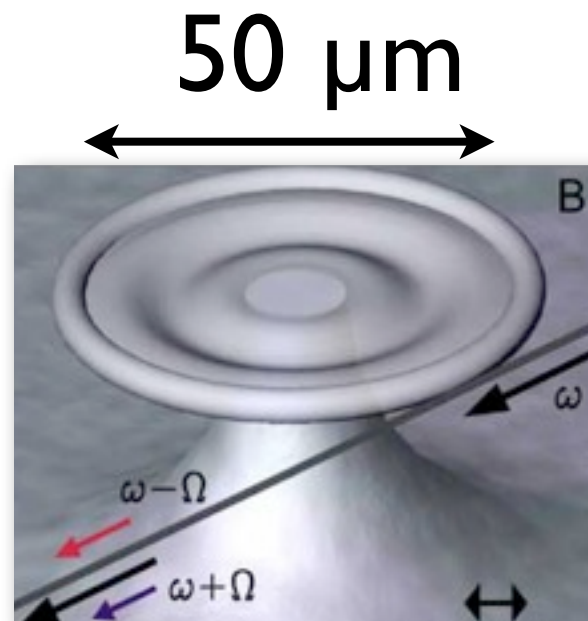
# Scaling down



# Scaling down



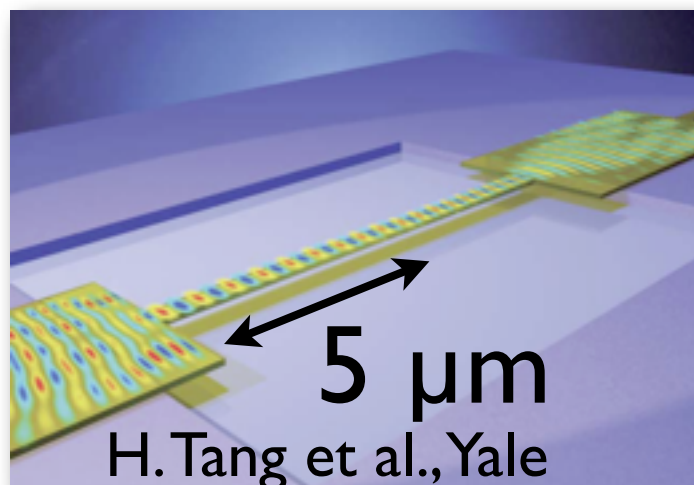
LKB, Aspelmeyer, Harris,  
Bouwmeester, ....



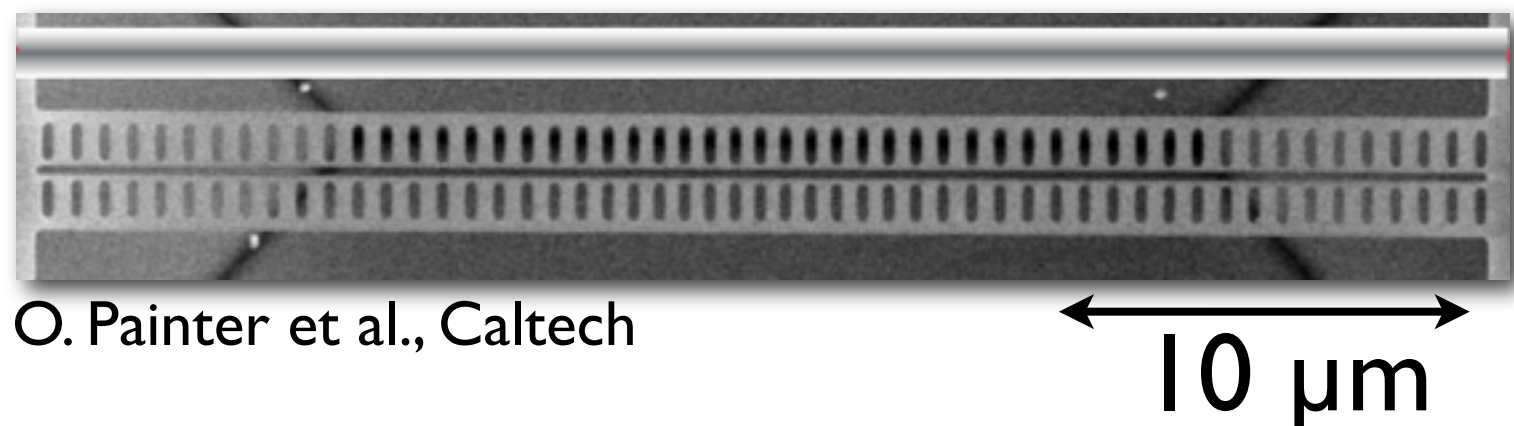
Vahala, Kippenberg, Carmon, ...

microtoroids

optomechanics in  
photonic circuits



optomechanical crystals



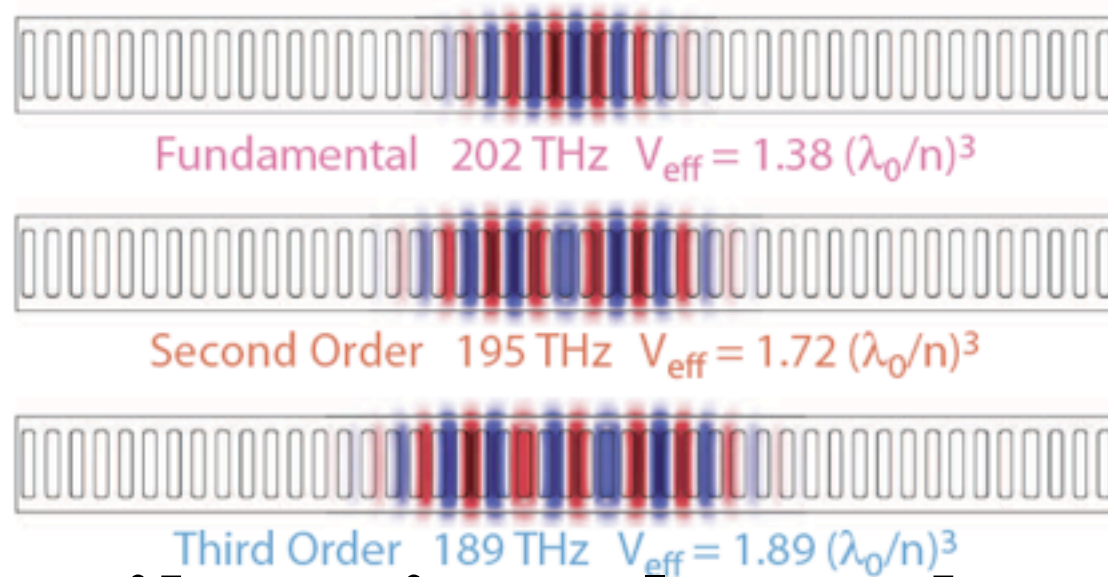
O. Painter et al., Caltech



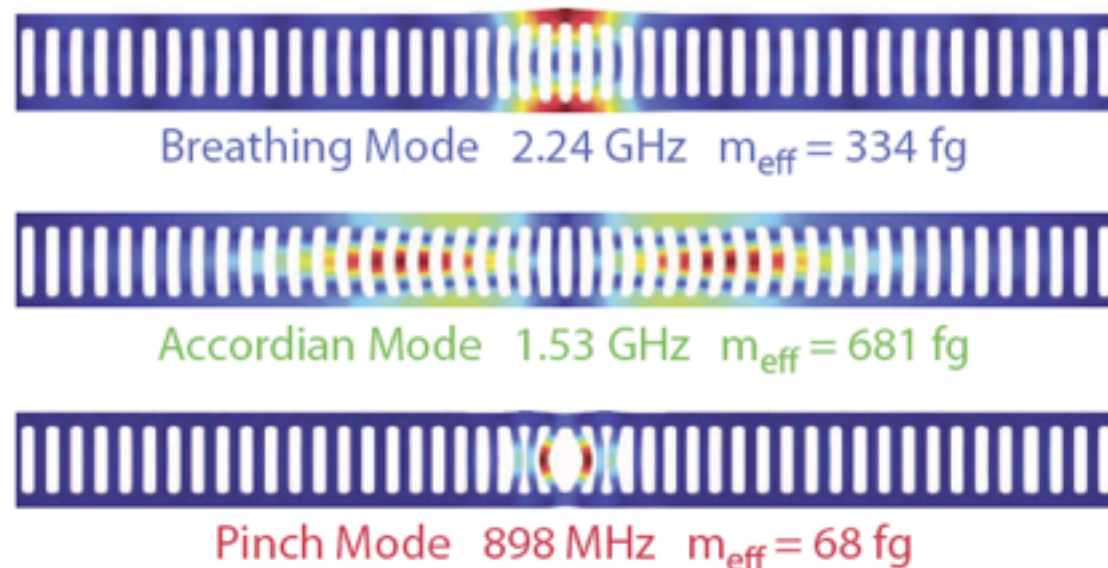
# Optomechanical crystals

## free-standing photonic crystal structures

### optical modes



### vibrational modes



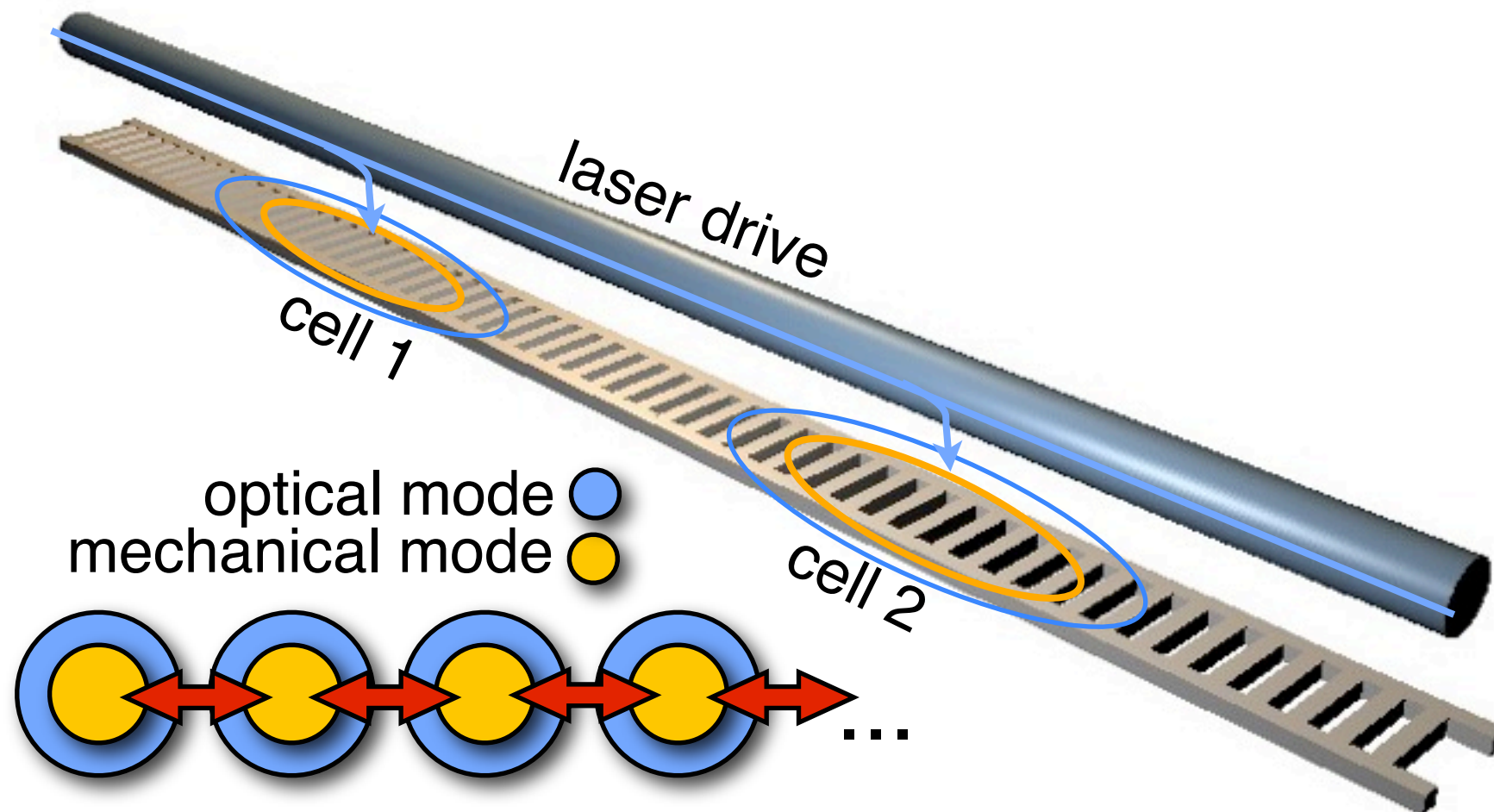
### advantages:

tight vibrational confinement:  
high frequencies, small mass  
(stronger quantum effects)

tight optical confinement:  
large optomechanical coupling  
(100 GHz/nm)

integrated on a chip

# Optomechanical arrays



collective nonlinear dynamics:  
classical / quantum

cf. Josephson arrays

# Dynamics in optomechanical arrays

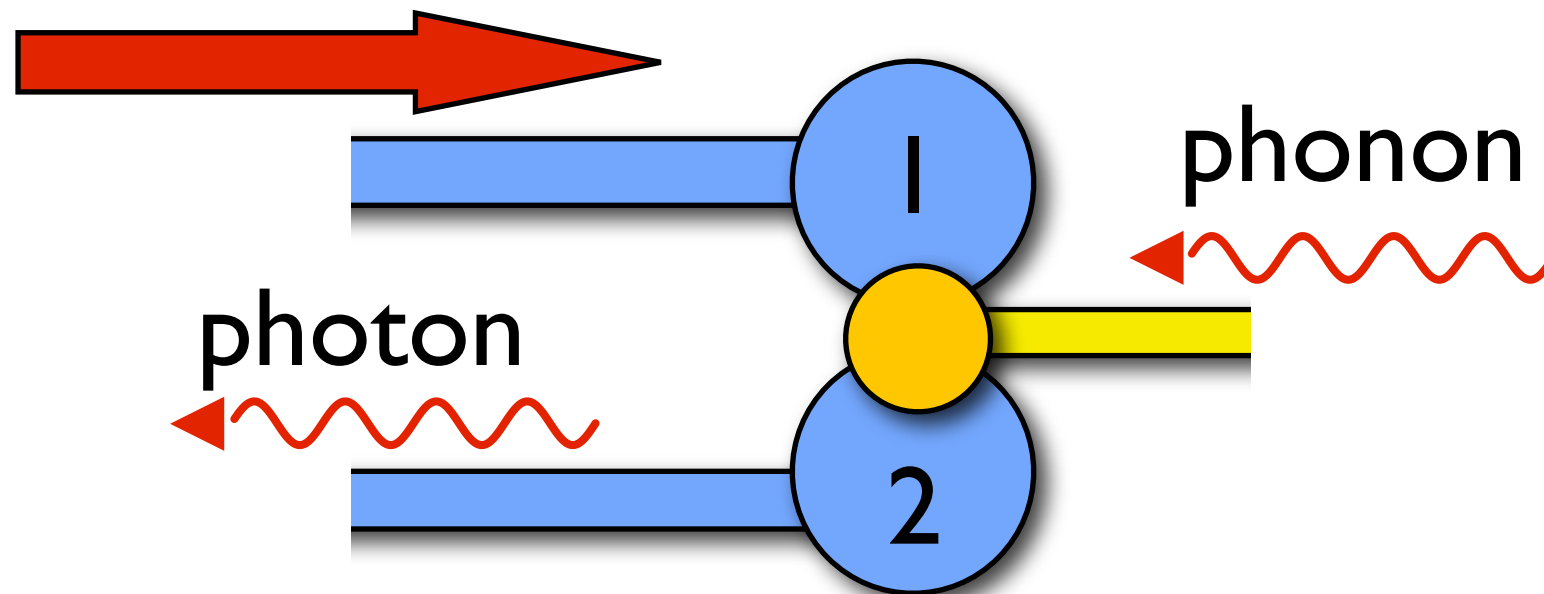
## Outlook

- 2D geometries
- Quantum or classical information processing and storage (continuous variables)
- Dissipative quantum many-body dynamics (quantum simulations)
- Hybrid devices: interfacing GHz qubits with light

# Photon-phonon translator

(concept: Painter group, Caltech)

strong optical “pump”

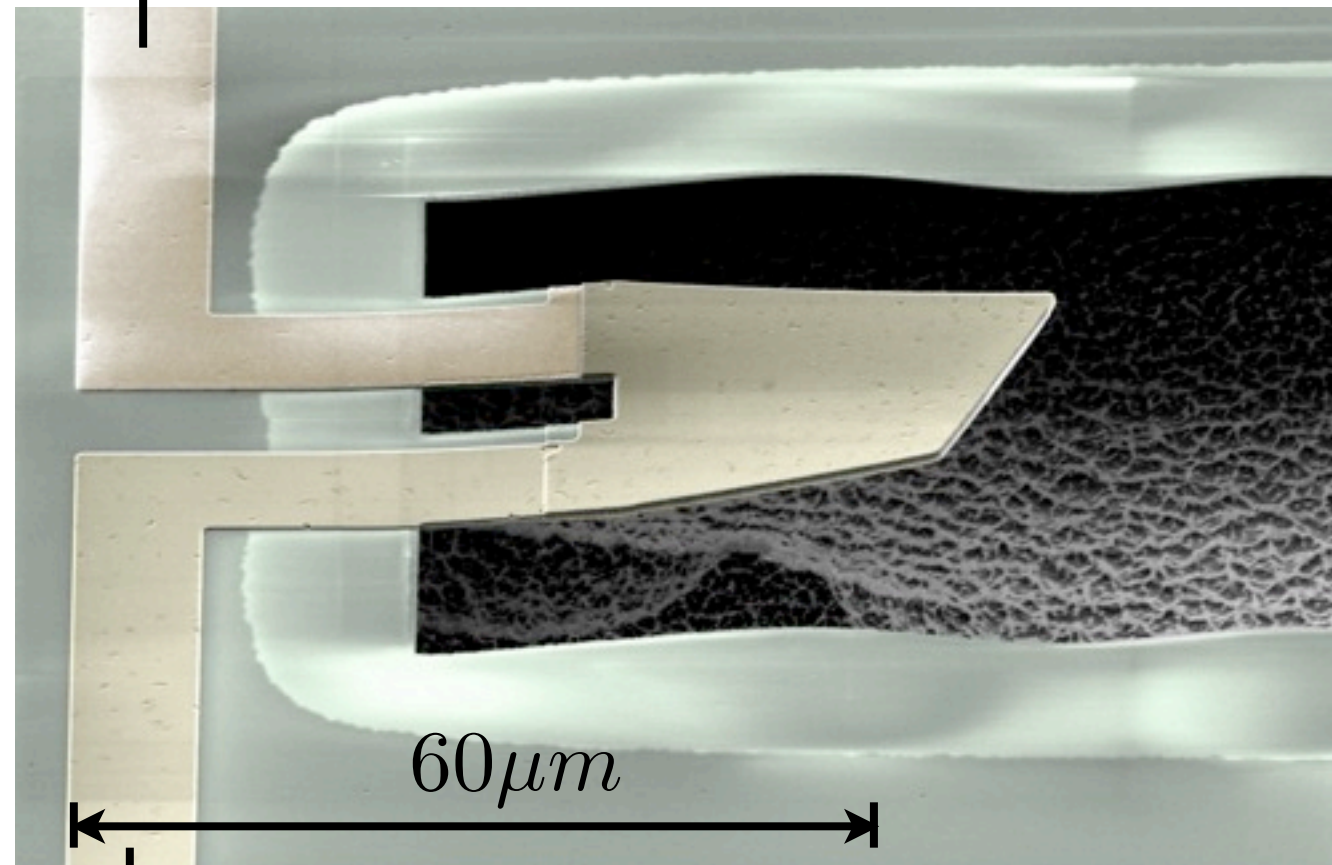


$$\hat{H} = \dots + \hbar g_0 (\hat{a}_2^\dagger \hat{a}_1 + \hat{a}_1^\dagger \hat{a}_2) (\hat{b} + \hat{b}^\dagger)$$

# Superconducting qubit coupled to nanomechanical resonator

2010: Celand & Martinis labs

**Josephson  
phase  
qubit**



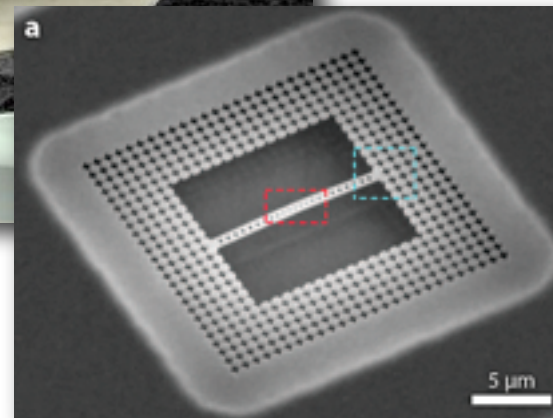
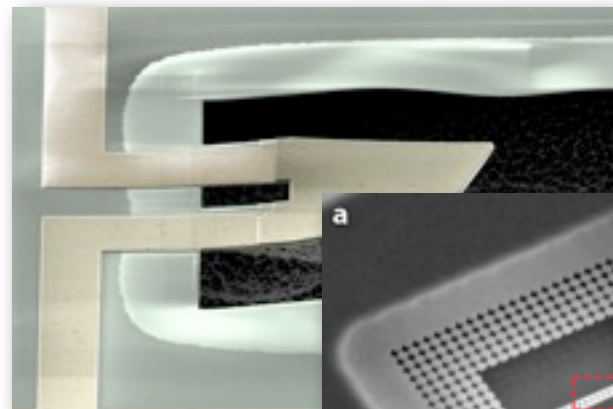
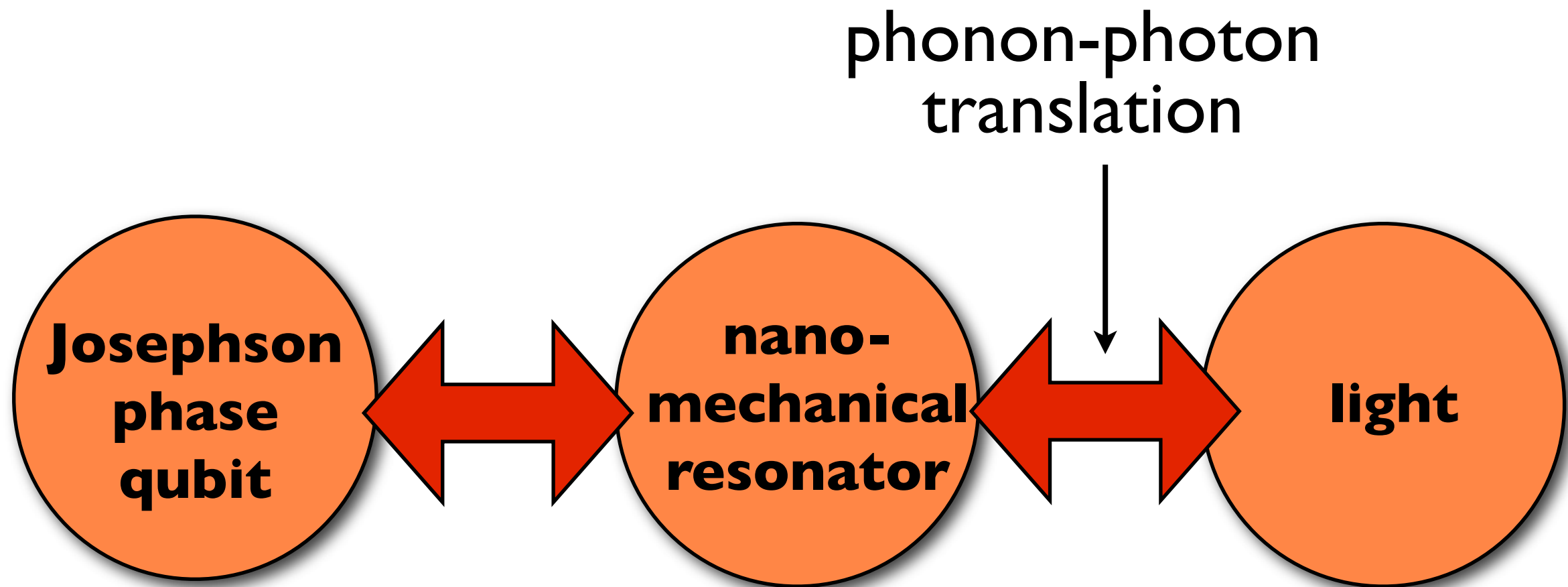
piezoelectric nanomechanical resonator

(GHz @ 20 mK: ground state!)

swap excitation between qubit and  
mechanical resonator in a few ns!



# Conversion of quantum information

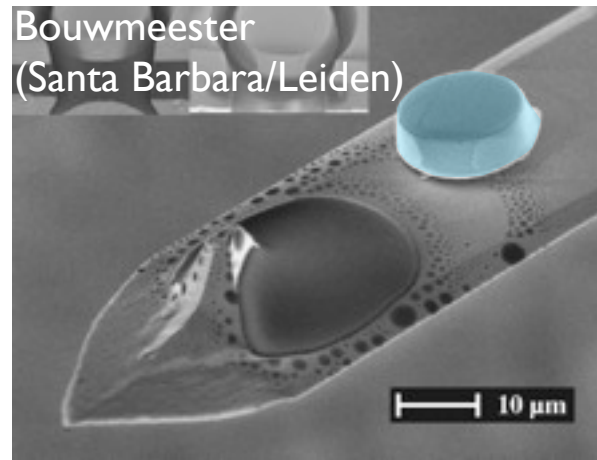


# Recent trends

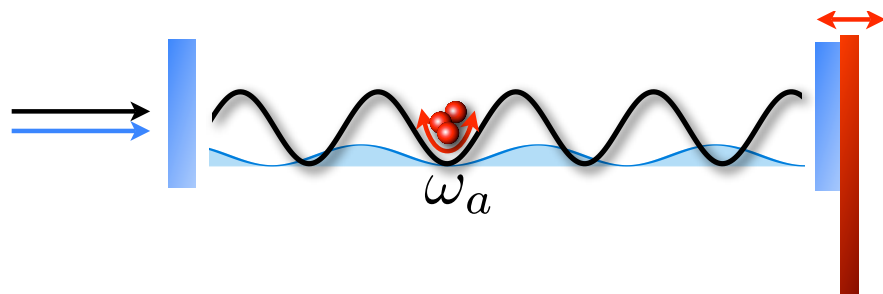
- Ground-state cooling: success! (spring 2011)  
[Teufel et al. in microwave circuit;  
Painter group in optical regime]
- Optomechanical (photonic) crystals
- Multiple mechanical/optical modes
- Option: build arrays or 'optomechanical circuits'
- Strong improvements in coupling
- Possibly soon: ultrastrong coupling (resolve single photon-phonon coupling)
- Hybrid systems: Convert GHz quantum information (superconducting qubit) to photons
- Hybrid systems: atom/mechanics [e.g. Treutlein group]
- Levitating spheres: weak decoherence!  
[Barker/ Chang et al./ Romero-Isart et al.]



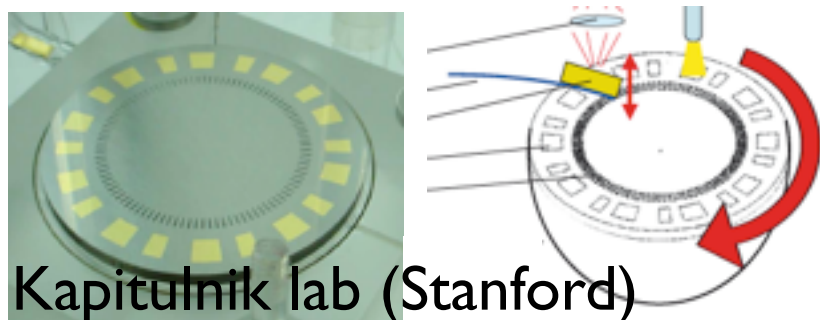
# Optomechanics: general outlook



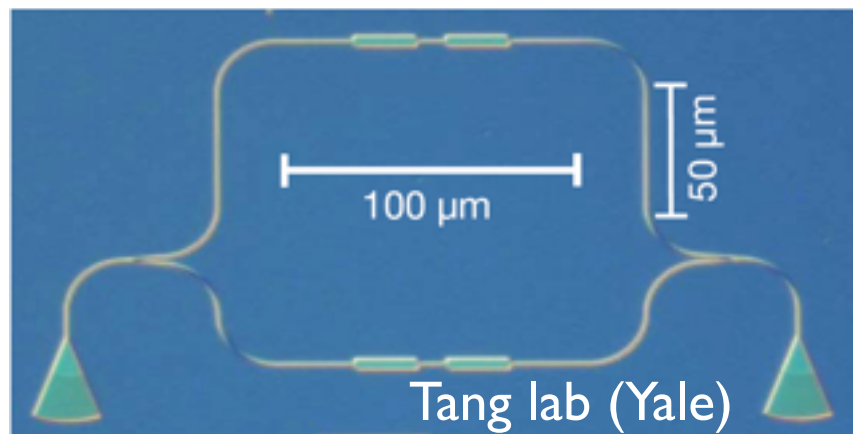
**Fundamental tests of quantum mechanics in a new regime:** entanglement with ‘macroscopic’ objects, unconventional decoherence? [e.g.: gravitationally induced?]



**Mechanics as a ‘bus’ for connecting hybrid components:** superconducting qubits, spins, photons, cold atoms, ....



**Precision measurements** [e.g. testing deviations from Newtonian gravity due to extra dimensions]



**Optomechanical circuits & arrays**  
Exploit nonlinearities for classical and quantum information processing, storage, and amplification; study collective dynamics in arrays

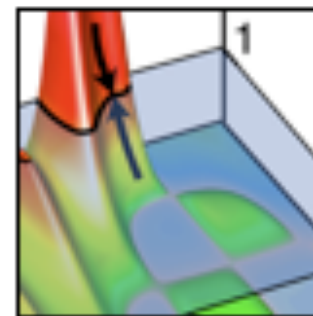
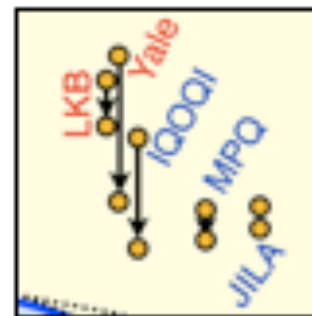
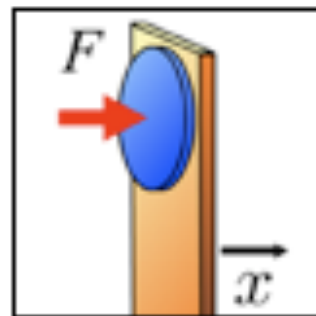
# Optomechanics

Recent review on optomechanics:  
APS Physics 2, 40 (2009)



## Trends

### Optomechanics



Florian Marquardt and Steven M. Girvin, May 18, 2009

Recent review on quantum limits for detection and amplification:  
Clerk, Devoret, Girvin, Marquardt, Schoelkopf; RMP 2010